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ON THE SURFACE TENSION OF LIQUIDS UNDER THE INFLUENCE OF ELECTROSTATIC INDUCTION.

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THE beginning of experimental inquiry into the relations subsisting between the capillary phenomena of a liquid and its state of electrification dates back to the middle of the last century. The subject has proved attractive to a host of physicists, and numerous investigations have been made and numerous phenomena described. Most of these phenomena may be referred to three general groups, according to the nature of their physical causes:

I. The first group includes those phenomena which are to be attributed principally to ordinary electrostatic forces of attraction and repulsion between the various (finite) portions of the liquid, or liquid and connected apparatus. Here belong, among others, the well-known phenomena of electrified jets and drops, investigated by Bose,[1] Nollet,[1] Beccaria,[2] Carmoy,[3] Singer,[4] Faraday,[5] Magnus,[6] Tate,[7] Beetz,[8] and Rayleigh;[9] the phenomena of electrified liquid

[1] Nollet, Reserches sur les causes des phénomènes électrique, 1749, seq. 327; Fischer, Geschicté der Physik, 5, 545 et seq.

[2] Beccaria, Dell. elletricismo artificiale, 1772.

[3] Carmoy, Obs. sur la Physique, sur l'Histoire Nat. et les Arts, 33, 340, 1788.

[4] Singer, Elements of Elec. and Electrochemistry, 1814.

[5] Faraday, Exp. Res., Art 1571, et seq.

[6] Magnus, Pogg. Ann. 106, 27, 1859.

[7] Tate, Phil. Mag. (4) 21, 452, 1861.

[8] Beetz, Pog. Ann. 144, 443, 1872.

[9] Rayleigh, Proc. R. S. L. 28, 406, 1879, et al.

films, observed by Van der Mensbrügghe,[1] Boys,[2] Kaiser,[3] and others; some of the effects attained by Herwig[4] with electrified mercury; and the phenomena of electrified falling drops, studied by Nichols and Clark.[5]

II. In the second group belong a great number of phenomena which are now known to be due principally to the presence of products of chemical change brought about by electrical action, and not primarily to this action itself. Many of these were once believed[6] to indicate a true effect of electrification upon surface tension, but their electrolytic nature is now established beyond doubt. Here are to be included the effects observed at the bounding surface between a liquid metal and electrolyte, when an external e. m. f., greater than their maximum e. m. f. of polarization is applied to them; and, if the views of one school of physicists are accepted (Cf. Group III), when any e. m. f., however small, is so applied. Effects of this kind were first observed by W. Henry[7] and by Gerboin;[8] later, by Hellwig,[9] Erman,[10] Draper,[11] and many others, including all who have worked on mercury-electrolyte potential relations with higher e. m. f.'s than the maximum previously mentioned. To this group belong also some of the effects obtained by Herwig.[12]

III. In the third and last group are included the phenomena with which we are here particularly concerned; those, namely, which are supposed to be caused by an actual change in the surface tension, due to the presence of an electric charge. By far the best known experiments dealing with this matter are those upon the capillary electrometer at low e. m. f.'s, and others the same in essential nature. It is well known that scientific opinion in regard to the nature of the phenomena here exhibited is divided between two theories and com-

[1] Van der Mensbrügghe, Pogg. Ann. 141, 287, 1870.
[2] Boys, Phil. Mag. (5) 25, 410, 417, 1888.
[3] Kaiser, Wied. Ann. 53, 671, 1894.
[4] Herwig, Pog. Ann. 159, 489, 1876; Wied Ann. 1, 73, 1877.
[5] Nichols and Clark, PHYSICAL REVIEW, 4, 375, 1897.
[6] Cf. Wied. Electricität, II., § 943, p. 737.
[7] W. Henry, Nichols. J. 4, 223; Gilb. Ann. 6, 370; 1800.
[8] Gerboin, Ann. de Chim. 41, 196; Gilb. Ann. 11, 340; 1801.
[9] Hellwig, Gilb. Ann. 32, 289, 1809.
[10] Erman, Gilb. Ann. 32, 261, 1809.
[11] Draper, Phil. Mag. (3) 26, 185, 1845.
[12] Herwig, loc. cit. ante.

binations of the two: the "Ladungsstromtheorie" of Helmholtz[1] and Lippmann,[2] and the "Leitungsstromtheorie" of Warburg[3] and Meyer.[4] If the latter theory, according to which the phenomena are wholly of an electrolytic nature, is correct, these phenomena must all be relegated to the second group above. But this theory, while it has gained considerable ground in recent years, has met with by no means general acceptance. The "Ladungsstromtheorie," according to which the phenomena are due wholly to electrostatic action, is still supported by very powerful names, including that of Ostwald[5] and that of Burch,[6] whose experiments, made with applied e. m. f.'s not greater than 0.5 volt, it seems impossible to explain by any form of electrolytic theory. An outline of the Ladungsstromtheorie is given by Meyer as follows:[7] "Nach. Hrn. v. Helmholtz wird die Polarisation des Meniscus im Capillarelectrometer als ein Ladungsphänomen aufgefasst. Zwischen Quecksilber und Electrolyt besteht ein Contactpotential-differentz, welche zur Entstehung einer electrischen Doppelschicht Anlass gibt, deren positiver Theil im Quecksilber liegt. Wird das Quecksilber kathodisch polarisirt, so vermindet sich die positive Ladung und die Oberflächenspannung nimmt zu, bis Gleichheit des Potentials zwischen Metall und Electrolyt besteht. Eine Polarisaation mit stärkeren Kräften ruft dann wieder durch die electrostatische Wirkung der in dem Meniscus vorhandenen negativen Ladung Verminderung der Oberflächenspannung hervor." This "electrostatische Wirkung" is described in the following words by Helmholtz himself:[8] "Jede elementare Elektricitätsmenge in einer solchen Doppelschicht wird abgestossen von den benachbarten gleichnamigen Mengen derselben Schicht, angezogen durch die entgegengesetzten der anderen Schicht. Da aber die Theile der

[1] Helmholtz, Berl. Monatsber., 945, 1881; Wied. Ann. 16, 30, 1882; Wiss. Abh. I, 925, 1882.

[2] Lippmann, Ann. de Chim. et de Phys. (5) 5, 515, 1875; et al.

[3] Warburg, Wied. Ann., 38, 321, 336, 1889.

[4] Meyer, Wied. Ann. 45, 508, 1892; 53, 845, 1894; 56.

[5] Ostwald, Allg. Chemie, 2 Aufl., II., 1, seq. 927, 1893; et al.

[6] Burch, Proc. R. S. L. 50, 172, 1891; Phil. Trans. 183 A, 104, 1892; and especially London Electrician, July 17–Aug. 21, 1896; and Proc. R. S. L. 60, 329–335, 1896.

[7] Meyer, Wied. Ann. 53, 867–868, 1894.

[8] Helmholtz, Wiss. Abh., I., 931 et. al. loc. cit.

eigenen Schicht näher sind, als die gleich grossen der entgegengesetzten, und näher den tangentialen Richtungen in der Fläche liegen, so wird die Abstossung in Richtung der Fläche die Anziehung überwiegen und in jeder mit einer Doppelschicht belegten Fläche muss die elektrostatische Kraft eine Dehnung der Fläche hervorzubringen streben. Wenn also die elektrisirte Fläche eine capillare Contractionskraft von gewisser Grösse hat, so wird die mit einer Doppelschicht beladene Fläche eine Verminderung der capillaren Spannung zeigen müssen. Es wäre also unter diesen Umständen zu erwarten, dass die capillare Spannung der Fläche im unbeladenen Zustande ein Maximum sein müsste." Whether the phenomena for which Helmholtz developed this theory are of electrostatic origin, as here conceived, or not, the reasoning on which the theory is based seems thoroughly sound when it is applied to the case of a liquid surface actually known to possess a static charge. The presence of the "Doppelschicht" and contact potential difference, which we have in the case of a metallic liquid and electrolyte, is not essential; an intense electrostatic field, however produced, effecting the same results. An immediate corollary of Helmholtz's theory is that the diminution of surface tension will be independent of the sign of the electric charge, and thus an even function of the electric surface density. Three investigations with reference to this effect have hitherto been made on surfaces statically charged to high potentials and must here be briefly discussed.

Herwig[1] in 1876 and 1877 described many experiments upon mercury surfaces. Among other results he found that a drop of mercury upon a surface of glass, ebonite, or other substance, became more flattened when highly electrified; that the liquid in a conical glass tube open at both ends and wider at the top, though supported by capillarity when unelectrified, fell through when charged; and that the depression of the mercury meniscus in a capillary tube of glass diminished considerably with electrification, the diminution being greater for positive than for negative charges. When the capillary tube, which was one branch of a U tube, was of iron, however, he observed no motion of the meniscus whatever, either for positive or negative charges, even when the meniscus

[1] Herwig, loc. cit. ante.

reached the end of the tube; while in similar experiments with glass tubes the mercury, when electrified, flowed out the end of the capillary branch. The experiments upon drops were repeated with various insulating and conducting substances as supporting surfaces, and under different conditions of the surface with respect to temperature and moisture. The experiments on glass tubes also were performed with the air above the meniscus in the capillary in various conditions; and in one set of experiments the tube was closed and was filled with pure hydrogen. From a study of all the phenomena observed Herwig concluded that the effects were due to several causes: (1) a diminution of the cohesion of mercury, (2) a change in the adhesion between mercury and glass, (3) oxydation of the mercury surface under the action of the positive charge, (4) a reducing effect of the negative charge, and (5) actual dissociation of the glass in contact with the charged mercury. The fact, however, that no effect was obtained with the capillary tube of iron, where chemical action, which in the glass capillaries and on the glass surfaces made itself manifest by a marked deposit, was prevented, would indicate that only an imperceptible change took place in the surface tension of pure mercury. Also, a consideration of the distribution of the Faraday tubes over the electrified surfaces of the mercury in the form of drops and in the conical tube shows that a part at least of the effects observed in these cases must have been due to ordinary electrostatic action.

In 1890 C. M. Smith,[1] at Edinburgh, applied the method of ripples to a study of the surface tension of mercury, making his measurements on photographs of the ripple-covered surface. The mean value of the tension of the unelectrified surface he found to be 529 dynes per cm. Two photographs were taken with the surface highly electrified with an induction machine, though no measurements of the electrification are given. The results were accordant and gave as the apparent[2] tension of the electrified surface only 421 dynes per cm. In view of experiments to be described later this diminution is extremely large, and great doubt is cast upon the result. If, as seems highly improbable, the effect was truly electrical the potential of the surface must have been enormous.

[1] C. M. Smith, Proc. R. S. E. **17**, 115, 1890.

[2] Smith conceived that the effect *might* be purely mechanical.

The last of the three investigations on this subject, in which the method of falling drops was employed, was published by Nichols and Clark[1] in 1897. With a given dropping tube the surface tension is proportional to the weight of a drop, provided cohesive and gravitational forces only act. When the falling drops and the apparatus are electrified, however, the mutual repulsion between the "elementary electric charges" of the surface, which may be included in the cohesive forces, as it is related in its effect with these only, is wholly obscured by the electrostatic repulsion between the various parts of the liquid drop and the tube; and the method is, therefore, incapable of giving determinations of the true surface tension effect.[2]

From the foregoing discussion of previous work it is obvious that to secure reliable measurements of this effect some method of investigating the surface tension must be adopted in the practice of which both electrolytic action and electrostatic disturbances—including all electrostatic action except that between the "elementary electric charges" at the surface—are excluded. Professor Nichols therefore suggested to me some months ago the investigation of this matter by a method which, though by no means completely excluding electrostatic disturbances, has proved more satisfactory for the purpose than those hitherto employed—the method of ripples in its latest and most nearly perfect form. That the sources of error mentioned are much reduced in importance through the use of this method will appear farther on.

The method of measuring surface tension now known as the method of ripples originated with a suggestion of Tait[3] in 1875 that Kelvin's[4] formula for the velocity of propagation over the surface of a perfect liquid of a train of plane sine waves of infinitely small amplitude might be utilized, since it contains the surface tension as an important factor, in the accurate determination of this constant. This formula has now been experimentally tested by a number of

[1] Nichols and Clark, loc. cit. ante.

[2] Cf. London Electrical Review, 40, 522, 1897; Houllevigue, Journal de Physique, 3, 6, 325, 1897.

[3] Tait, Proc. R. S. E. 8, 485, 1875.

[4] W. Thomson, Phil. Mag. (4), 42, 375, 1871.

investigators, including Matthiessen,[1] Rayleigh,[2] Dorsey,[3] and others; and the method based on it has been shown to be capable of giving extremely accurate results in close agreement with the best of those obtained by other means. The great advantages of this method over others, so far as the determination of the tension of unelectrified surfaces is concerned, lie in the fact that with it the measurements are all made upon the liquid surface only and are thus independent of all contacts and contact angles between liquid and solids, and in the means it affords of using surfaces of the highest degree of purity. For the detailed history of the method and the discussion of the effects of viscosity, finite amplitude, floating dust particles, etc.—causes of deviation from the conditions of Kelvin's formula—reference must here be made to the papers of previous investigators.[4]

If[5] we denote by g the acceleration of gravity, ρ the density of the liquid, h its depth, λ the wave-length, n the wave frequency, T the surface tension, and V the velocity of propagation of the waves, Kelvin's equation is

$$V^2 = n^2\lambda^2 = \left(\frac{g\lambda}{2\pi} + \frac{2\pi T}{\rho\lambda}\right) \tanh \frac{2\pi h}{\lambda};$$

whence

$$T = \rho\left(\frac{\lambda^3 n^2}{2\pi} \coth \frac{2\pi h}{\lambda} - \frac{g\lambda^2}{4\pi^2}\right).$$

The quantity $\coth \frac{2\pi h}{\lambda}$ was in these experiments sensibly equal to unity. Even with λ so large as 0.5 cm. and h so small as 0.3 cm., which limits were not reached, $\coth \frac{2\pi h}{\lambda}$ differs from unity by only one part in about 1900; for $h = 0.4$ cm. and λ the same as before, by less than one part in 23,000. The determination of T in absolute measure is thus reduced to the measurement of λ, ρ, n, and g.

Kelvin's formula postulates, however, that the only forces acting are cohesion and gravitation; and it cannot therefore be used, with-

[1] Matthiessen, Wied. Ann. 38, 118, 1889.

[2] Rayleigh, Phil. Mag. (5) 30, 386, 1890.

[3] Dorsey, PHYSICAL REVIEW, 5, 170 and 182, 1897.

[4] Cf. especially the papers of Rayleigh and Dorsey, loc. cit. ante, and Tait, Proc R. S. E. 17, 110, 1890.

[5] Cf. Rayleigh, loc. cit. ante.

out modification, to find the *true* surface tension of an *electrified* surface, but will give only the *apparent* surface tension, determined from the wave-length and the constants of the equation. The wave-length, however, is affected not only by the change (if any) in the cohesion due to electrostatic induction over the surface, but also by the other kinds of electrostatic action present. All but one of these disturbances may be safely neglected as affecting the wave-length by only insensible amounts. One of them, however, can be by no means disregarded: The effect of the electrostatic field is to diminish the potential energy of the deformation of the surface due to the wave motion, and thus to diminish the velocity of the waves, corresponding to a given frequency or to diminish the wave-length—an effect in the same direction as that due to a diminution of surface tension. The *true* surface tension, at any state of electrification, will therefore be found from the formula by adding to the second member, which gives the *apparent* surface tension, an intrinsically positive function of the square of the electric surface density:

$$T = \rho\left(\frac{\lambda^3 n^2}{2\pi}\coth\frac{2\pi h}{\lambda} - \frac{g\lambda^2}{4\pi^2}\right) + F(\sigma^2).$$

F is an even function of the surface density (σ) since the electrostatic effect it represents must be independent of the sign of the charge. The function vanishes with σ. This formula will be returned to later.

For the production and measurement of ripples, Lord Rayleigh's[1] experimental method was adopted nearly in the elegantly modified form used by Dorsey.[2] Rayleigh's method consists essentially in the production of waves satisfying as nearly as is practicable the conditions of Kelvin's formula; and in the measurement of their wave-length by the aid of light made intermittent in the period of the waves and by the employment of Foucault's optical method for rendering visible small departures from truth in plane or spherical reflecting surfaces.[3] While Dorsey's disposition of apparatus has been in general followed, some improvements have been made, as

[1] Rayleigh, loc. cit. ante.

[2] Dorsey, loc. cit. ante.

[3] Cf. Rayleigh, Theory of Sound, II., 345.

well as alterations better adapting the apparatus to the requirements of the present work. For the sake of completeness and of these modifications a description will now be given.

The liquid to be experimented upon was contained in a shallow vessel N (Fig. 1), which was supported on the massive stone slab X. On this slab was also carried, at a height of about 20 cm., and in adjustable mountings, the large Kŏnig fork F_2, supplied with sliding weights, etc. To its lower prong was clamped by means of a "counter weight" the dipper D, which generated the ripples on the surface of the liquid, into which its glass plate lightly dipped. Several dippers were used during the investigation; the construction of the one used in the principal experiments is shown in Fig. 2. G was a wedge-shaped glass plate of the dimensions indicated, connected by the slender hard rubber rod R, fastened with shellac at both ends, to the stiff aluminium strip A, which was clamped directly to the fork. In the experiments on water the portion P of the plate G was coated with paraffine, and in those on mercury, was left uncoated. The rest of the plate was coated with shellac, both this and the paraffine being used to prevent creeping of the charge. The ripple fork F_2 (Fig. 1) was driven electro-magnetically by a shunt circuit from the terminals of the similar fork F_1, with which it was tuned into unison and which was also driven electro-magnetically (platinum contact). The frequency of F_1 was determined by an arrangement similar to Lissajous' "mirror vibroscope" but simpler. The horizontal motion of a polished steel ball attached to one prong of F_1 near B and illuminated by the lamp I_2, combined with the motion, at right angles to that of the ball, of the mirror M_3 of the massive standard fork F_3, gave a Lissajous' figure in the focal plane of the telescope T_2. The temperature coefficient of the frequency of F_3 was known. This apparatus was set up early in the course of the work when large effects were expected and some absolute measurements contemplated. In view of the small effects actually observed the frequency of F_1 could probably have been obtained with sufficient accuracy by a moderately good ear without comparison with F_3. The frequency of F_1, in use, was 64.4.

The ripples were illuminated by the 20 c. p. incandescent lamp I_1, the image of whose *thick* filament was formed by the lens L_1

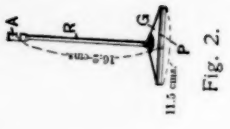


Fig. 2.

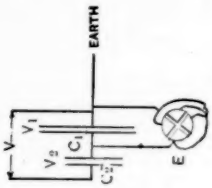
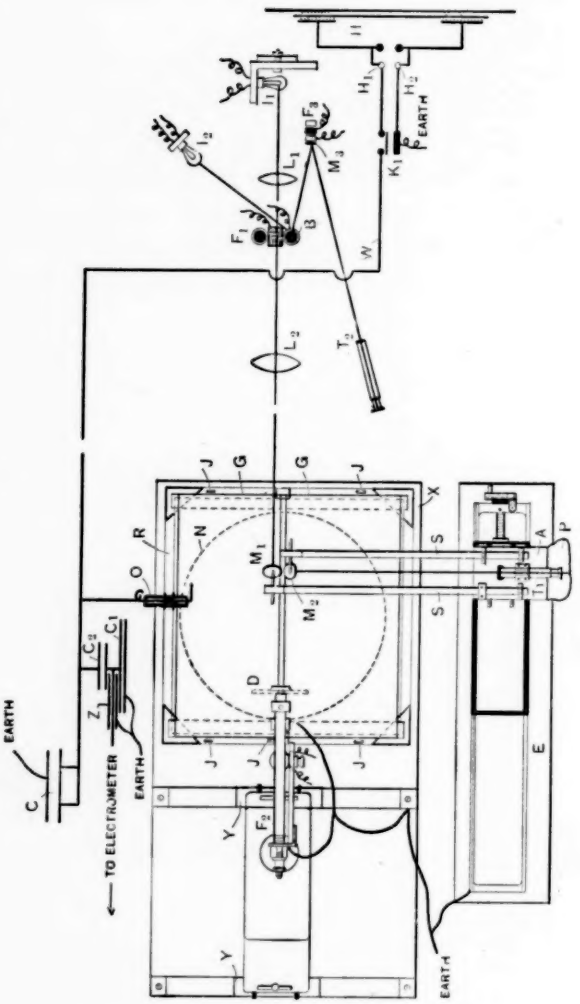


Fig. 3.



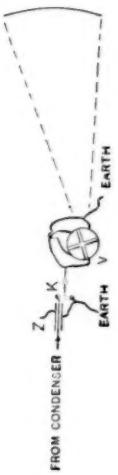


Fig. 1.

upon two stiff aluminum vanes clamped firmly by the counter-weights to the prongs of F_1. They overlapped between the prongs and were pierced by a very small circular hole, through which the light converged by L_1 passed when the prongs were in their mean positions. The hole was at the principal focus of a second lens L_2, after passing through which the light formed a parallel beam. At a distance of more than a meter from L_2 the light struck the mirror M_1, was reflected by it to the liquid surface, thence to a second mirror M_2, and after a third reflection passed into the observing telescope T_1. M_1 and M_2 were large König fork mirrors with plain metallic faces and were so attached to the heavy glass tubes SS that each could be rotated about one axis perpendicular to its supporting tube, and could be moved for a few centimeters in the direction of this axis. The tubes SS (over 60 cm. long) were clamped to the wooden block A, and they also could be rotated about their axes and moved in the directions of their lengths. The telescope T_1, provided with adjusting screws, and the counter-balancing weight P of sheet lead were also mounted upon the block A. The cross hairs of the telescope and the centers of the two mirrors were nearly in the vertical plane passing through the direction of the waves. The wooden block was rigidly clamped to the carriage of the dividing engine E. The engine was provided with levelling screws and supported on one end of a firm table resting squarely on the cement basement floor and projecting underneath and beyond the stone slab X.

With[1] the apparatus in proper working order and in action the field of the telescope focused for the crests of the waves reveals a series of bright bands, one coming from each crest by Foucault's process. They appear stationary, since the period of the ripple fork is identical with that of the fork which renders the light intermittent, being driven by it even when not tuned in unison. The bands are separated by distances corresponding approximately to half a wave-length, since the light is intermittent with twice the frequency of the forks. The waves are not perfectly symmetrical in their two halves, or the crests would appear exactly half a wave-length apart. The forks were tuned to unison by moving the slid-

[1] Cf. Dorsey, loc. cit. ante.

ing weights on F_2 until the bands in the field remained stationary when the circuit driving F_2 was broken. This insured the stationary appearance even when the platinum wire failed to make perfectly regular contact. To measure the wave-length the cross hairs of the telescope were set upon one or more crests by means of the screw of the engine and then upon one or more a whole number of wave-lengths nearer the fork. From the readings of the screw and the number of waves passed over the wave-length at once follows.

That measurements by this method be reliable, it is necessary, first of all, that the wave pattern be steady and that the waves be straight and parallel. The first condition was satisfied by mounting the tuning fork F_2 and the vessel containing the liquid upon rubber tubing in the manner of Rayleigh, and by a method of suspension somewhat improved over one which has been in use here for a number of years[1] and which is similar to that used by Dorsey. The heavy stone slab X was used, and was mounted on a slightly larger wooden panel suspended from the ceiling by twisted cords attached at three points and very roughly parallel. The cords were previously soaked in melted paraffine to prevent changes in length, due to variations in the hygrometric state of the atmosphere. When the apparatus hung freely no extraneous vibrations appeared to affect the surface except those due to walking on the floor overhead. During the electrical experiments a few tufts of cotton waste were placed loosely between the wooden panel and the table to prevent any possible motion due to static charges, but extraneous disturbances, again excepting walking overhead, gave practically no trouble. As to the second point, the form of dipper used insures sufficiently straight waves when it is clean and the surface is in proper condition. It is easily cleaned, when the waves have ceased to be straight and parallel, by rubbing the paraffine with clean filter paper, and treating the glass surface—when it is used—by ordinary processes of cleansing. It is not necessary that the dipper be wetted by the liquid, but only that the part of it coming in contact with the surface be uniform throughout and free from anything which can alter the surface. It should be remarked here also that the deviation of the waves from sine form, before referred to, and from infinitely small amplitude,

[1] Due to O. M. Stewart and described by him in THIS JOURNAL, 4, 442, 1897.

do not appreciably affect the wave-length when the amplitude is very small. That this statement is correct, at least when relative measurements are concerned, there is abundant evidence, both from experiments made by Dorsey and from some performed during this investigation. The amptitudes here used were so small that the ripples could be seen only by some stroboscopic means.

Several other conditions must be satisfied:[1] (1) The mountings of the mirrors, etc., must be rigid—which is easily secured. (2) The screw of the engine and its ways in the parts used must be sensibly correct; the screw was assumed to be without sensible error from a knowledge of the general excellence of the engine, and the ways were tested by Dorsey's second and more accurate method, without discovery of any irregularities. (3) The light must issue from L_2 in a beam which is parallel at least vertically; the position of F_1 was adjusted until the vertical diameter of the beam was the same (5.5 cm.) just beyond L_2 and at a point a meter and a half further on. It is not necessary that the beam be horizontal or parallel to the ways of the engine provided it is of sufficient cross-section otherwise to illuminate M_1 in all its positions. (4) In the case of absolute measurements the ways of the engine must be parallel to the liquid surface and perpendicular to the vertical plane through the direction of the ripples. Both conditions are easily satisfied, as the engine and ripple fork being in adjustable mountings; though in the case of relative measurements neither adjustment is of importance. (5) To avoid error due to the viscosity of the liquid and consequent damping of the waves, the light producing a band in the field of the telescope should come from as small a strip as practicable in the immediate neighborhood of a crest, and the light should be thrown symmetrically by M_1 on both sides of the crest. This was practically effected by the arrangement of telescope and mirrors described.

The electrical part of the apparatus is also shown diagrammatically in Fig. 1. In default of the ideal instrument, a high potential battery, a Holtz machine H, driven by an electric motor, was used to produce electrification. Contact with the liquid, the steadying capacity C, consisting of a few Leyden jars, and the condenser C_2,

[1] Cf. Dorsey, loc. cit. ante.

which formed part of the electrometer arrangement yet to be described, was made through the key K_1 by the large smooth copper wire W, carried from the machine on rubber rod supports. One pole of the Holtz machine, one coating of the condenser C, the dividing engine, the ripple fork, and the stone slab X were grounded. The vessel containing the liquid was insulated from X by an abundant supply of glass. In the experiments upon water it was a circular copper dish 51 cm. in diameter and not over 1.3 cm. deep; in those upon mercury, a rectangular box of wood of inside dimensions 37 by 44 by 1.5 cm. The Holtz machine and key K_1, were separated from the ripple apparatus and electrometer by the wall of a small room in which they were kept. When the key was placed near the ripple surface even the slight mechanical disturbance due to its operation was communicated by the wire to the surface and injured the observations.

According to the results of previous investigators the diminution of surface tension due to electrification is so large that it would be easily possible, without the use of a measuring instrument, to detect, with the aid merely of stroboscopic vision, the corresponding alteration in the length of the waves. This was tested as follows: A dipper was constructed with its glass plate so long as to produce waves parallel and straight for a length of 16 cm. or more. The copper dish was nearly filled with water and so placed that one end of the dipper was near its edge. The straight portions of the waves then extended from near the edge to near the center of the vessel—a range over which the electric density of the charged surface varied greatly. The surface was illuminated by a window and observed through the openings of a rotating disk driven by a motor with such speed that the wave pattern was steady enough to be clearly seen. If the effect sought were large the wave-length would be seen to diminish under electrification and the waves would cease to be parallel, the diminution being greater near the edge than near the center of the surface. On electrifying the liquid to a potential of about 12,000 volts no change whatever was detected.

A series of accurate measurements of the wave-length on surfaces of both water and mercury was then begun. Four successive crests were usually set upon at a distance of about 15 cm. from

the dipper and four more 20 wave-lengths nearer, thus giving four independent determinations of the wave-length for one complete observation. One or more such observations were made with the surface uncharged, the time of each being noted, then one or more in precisely similar manner with the surface charged to a potential usually in the neighborhood of 25,000 volts. The process was repeated a number of times for each surface examined. Several specimens of mercury and one of water were examined in this way, and the results are exhibited in the curves of Fig. 4 with wave-lengths as ordinates and times as abscissæ. The average acci-

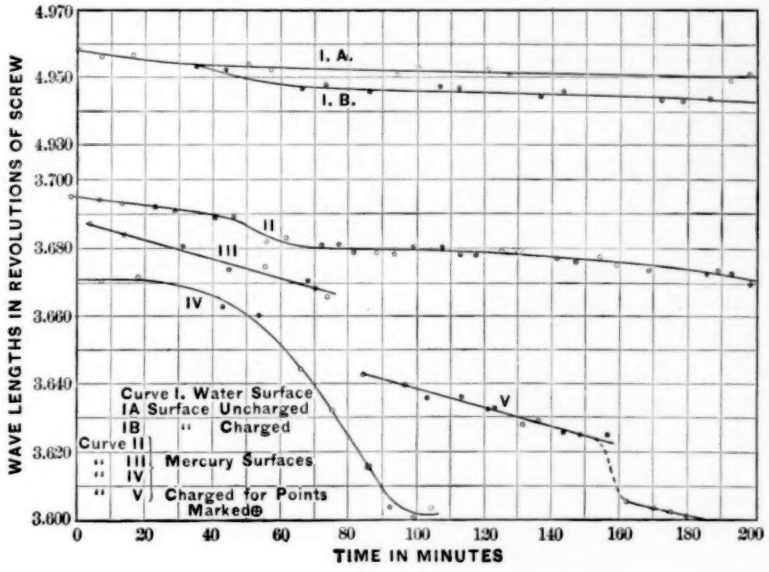


Fig. 4.

dental error in measuring the wave-length was less than one part in 4,000 in the case of mercury, and less than one part in 6,000 in the case of water. The crosses in circles and circles denote measurements upon charged and uncharged surfaces respectively. No change in the case of mercury was detected, indicating that the change in the apparent tension was not so great as three parts in 4,000 $\left(\frac{\Delta T}{T} = \text{approx. } 3\frac{\Delta\lambda}{\lambda}\right)$. In the case of water, however, there was a

distinct, though very small, effect, the time curve splitting up into two and indicating a diminution of the wave-length due to electrification. The gradual diminution of wave-length with time must be attributed to changes of temperature, contamination of surface, etc. Some of the mercury surfaces were good, others poor, the better giving always the greater wave-length. The tension of mercury, as the curves show, is subject at times to very rapid alterations; probably due in these cases to rapid contamination.

From these experiments it is evident that to obtain effects measurable with any accuracy much greater intensity of electric field over the surface must be produced. This field must also be as nearly uniform as possible. The following way of effecting both these results was adopted: The liquid surface was made the lower coating of a horizontal parallel plate air condenser. The other coating, which was grounded, was of tin foil fastened smoothly with shellac to the under surfaces of two heavy plane plates of glass *GG* (Fig. 1). These plates rested on narrow plane glass supports mounted by means of glass corner pieces upon two sides of the rigid, plane, rectangular wooden frame *R*. The plates *GG* were adjusted to a distance of from 8.5 to 10 or 11 mm. apart, and their inner edges, over which the tin foil continued, were made parallel to the ways of the engine. The narrow slit thus formed admitted the dipper rod *R* (Fig. 2), and also permitted the passage of all necessary light. The area of the tin foil was somewhat greater than that of the liquid surface. A glass tube *O* (Fig. 1) admitted the wire from the Holtz machine. The frame was provided with leveling and supporting screws *JJJJ*. By means of these screws and a spirit level mounted on one of the plates *G*, whose thickness was uniform, the two surfaces, liquid and tin foil, were rendered parallel and kept so. The distance between them was measured with a micrometer constructed for the purpose and consisting of a long screw turned to a point at one end and passing perpendicularly through a plane brass plate. To perform the measurement the plate was placed at the edge of the slit, with the pointed end of the screw projecting toward the liquid, and the screw was turned until the point and its image in the surface came in contact. The distance from the point to the brass plate was then

measured, and the distance between the condenser plates obtained from this by substracting the thickness of the glass plate and foil. As the waves measured extended over a distance always less than 12 cm. in the central portion of the surface, while the distance between the liquid surface and the tin foil was always less than 4.1 cm., practical uniformity of electric field was secured. The effect of the width of the slit was considered negligible.

The only measurement now remaining to be discussed is that of potentials. As high potentials were of necessity employed and no reliable high potential instrument was on hand, it was necessary either to construct such an instrument or to devise some means by which a low potential electrometer could be used to measure very high potentials. The latter course was chosen and some methods devised by which the range of an electrometer or ballistic galvanometer may be almost indefinitely extended. The first method which suggested itself is an inversion of Faraday's well-known method of comparing capacities. The ratio of capacities being known, the initial potential can be computed from the measurement of the final, which may be made as low as desired by suitably selecting or adjusting the condensers to be used. The second method, which was used in the investigation, consists in applying the whole potential difference to the terminals of two condensers arranged in series and measuring the potential difference between the plates of one of them. From this and the known ratio of capacities the whole potential difference follows. Let C_1 and C_2 (Fig. 3) denote two condensers and their capacities connected in series and having the joint capacity C. Let the electrometer E be connected to the coats of C_1, the terminal coat of which is earthed. Let V be the whole fall of potential through the series and V_1 that through C_1. Then we have, including in C_1 the capacity of the electrometer,

$$CV = C_1 V_1 = \frac{C_1 C_2}{C_1 + C_2} V; \text{ whence } V = V_1 \frac{C_1 + C_2}{C_2}.$$

By suitably adjusting the ratio $\frac{C_1 + C_2}{C_2}$ an electrometer (or ballistic galvanometer) of any convenient range may be used to measure any potential within its range or higher—except in so far as leakage, ab-

sorption, etc., make the indications of the instrument unreliable. In the first described method these difficulties are wholly or almost wholly avoided. From the above equation we have, solving for the ratio of capacities,

$$\frac{C_1}{C_2} = \frac{V - V_1}{V_1}.$$

Thus by measuring both V and V_1, which may easily be done with the same electrometer when a constant battery is used, capacities may be very accurately and expeditiously compared.

The electrometer arrangement actually used is shown in Fig. 1. The electrometer V was a Thomson quadrant, used idiostatically, and read up to 400 volts. The quadrants were adjusted to accurate symmetry, and the same deflections were obtained for positive and negative charges of the needle. The jar was connected in and the case, induction plate, and its pair of quadrants grounded. The other pair of quadrants was connected to the insulated coating of C_1, through the key K, by an insulated wire running through the grounded leaden tube Z for protection from static disturbances. C_1 was a mica condenser, and C_2 a pair of leyden jars in multiple. The jars were insulated on glass varnished with shellac, and connections were made as shown in the diagram. The ratio of capacities $\frac{C_1 + C_2}{C_2}$ was 75 to 1. The capacities were determined by the method above indicated, comparisons being made with a Nalder standard. The capacities of the electrometer and connecting tube were negligible in the above ratio. In actual practice with high potentials the leakage and absorption of the condensers are very troublesome. That the electrometer may give reliable indications, the ratio of capacities, including the absorption and leakage, must remain the same during the observations as the ratio of calibration. This, with the condensers used, was not the case except when the condensers and electrometer—which will together be referred to as the condenser electrometer—were charged rapidly. It was then approximately true as the following test shows: The condenser series was short circuited, and the electrometer as quickly as possible thereafter insulated by means of the key K. The spot of light came to rest some-

times at the zero of the scale and sometimes a few divisions from zero. This test was made repeatedly and indicated that the readings averaged a few per cent. too low. That they were too low and not too high, follows from the fact that the charge remaining after short circuiting was always of sign opposite to that of the original charge. Whether this method of measuring potentials is new or not I do not know, but I have not succeeded in finding any account of it published. Since it occurred to me, however, I have found that it had previously occurred also to other members of this laboratory, and its simplicity is so great that it can hardly fail to have occurred to others elsewhere. In addition to the two condenser methods here given for measuring potentials, any electrometer method of measuring capacities may be inverted for the same purpose.

As a desirable check upon the indications of the condenser electrometer, it was used to measure the potentials corresponding to a number of spark lengths between polished brass spheres 3 cm. in diameter. The mean values of the spark lengths and electrometer potentials are given in Table I., together with Baille's[1] potentials for the same spark lengths between spheres of the same diameter. The measurements of spark lengths were only rough. In Fig. 5 the results of the comparison are shown graphically. The most reliable determinations with the condenser electrometer were the last two, which agree closely with Baille's results. If the agreement were exact the continuous and broken lines would coincide.

With the apparatus as now described an extended series of observations was made upon surfaces of water and mercury. Since the indications of the electrometer, as before stated, were reliable only when the instrument was rapidly charged, the following method of procedure was adopted: With the surface unelectrified and all the condensers discharged, two crests A and B, a known number of wave-lengths apart and equidistant from the center of the surface, were chosen and the position of one of them, say A, accurately determined by several settings of the cross hairs of the telescope. Through the key K_1 connection was then made with the Holtz machine. The electrification—the sign of which was readily determined by observation of the brushes given off at the discharging

[1] Baille, Ann. de Chim. et de Phys. (5) 25, 531, 1882.

TABLE I.

Electrometer Test.

Spark length in cms.	Potential in volts; Baille's values.	Potential in volts as measured with condenser electrometer.	No. of observations with condenser electrometer.
0.2	7,800	6,200	2
0.42	14,530	12,800	4
0.50	16,500	15,200[1]	—
0.67	21,450	20,400	10
0.82	25,910	26,200	3
0.93	29,290	29,400	3

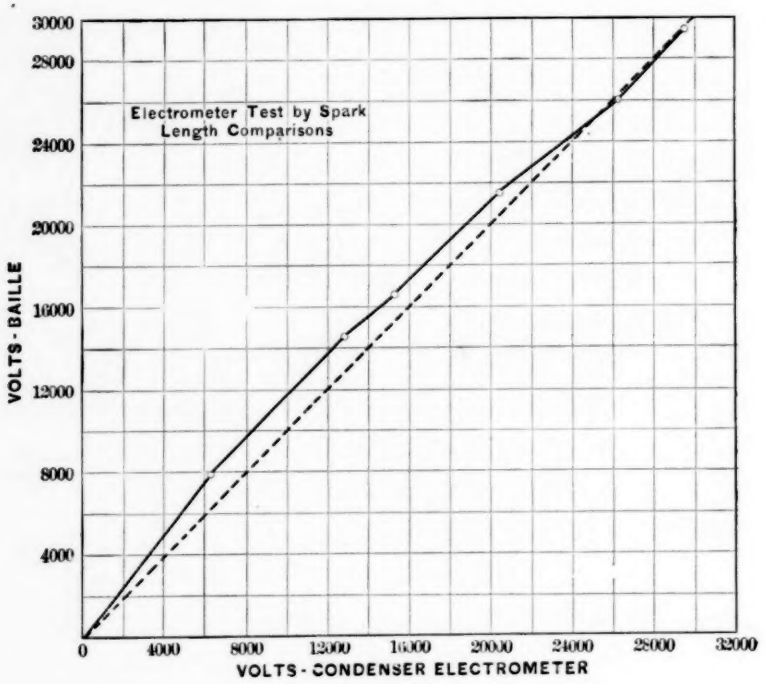


Fig. 5.

points to the plate of the machine—rose and the crest moved toward the fork F_2. The cross hairs were again set upon it and the deflection on the scale of the electrometer—which was so placed

[1] This value was obtained by interpolation.

that the observer at the telescope could read it by merely turning his head—at once noted. The reading of the screw of the engine was then taken and the process rapidly repeated until the electrometer indication began to fall, showing that it was not safe to proceed farther. The process of electrification was very often made so rapid that only one or two observations could be taken in a set. The condenser electrometer then ceased to have any advantage over the electrometer based on Faraday's method of comparing capacities—was in fact at a disadvantage owing to the presence of absorption and leakage. After each set the condensers, including the liquid surface and tin foil, were all discharged through the key K_1. The two condensers, C_1 and electrometer, and C_2 were by this put in multiple and, still retaining a charge on account of absorption, etc., were then discharged. The whole process was then repeated once or more and the crest B observed in the same manner. The sign of the electrification was changed at will by interchanging the connections at the terminals H_1 and H_2 of the induction machine. Until it was certain that experimental conditions could be kept practically constant, the interchange of the signs of the electrification, and of the crests whose motion was measured, was frequent.

To avoid dampness in the air and consequent poor insulation, the experiments were performed at the high temperature of 22°–23° C.

In the experiments upon water clean tap water was used and the observations were made upon two surfaces. Conditions were kept so nearly constant that the wave-length for the unelectrified surface changed during the first set only from 4.959 R to 4.954 R, and during the second set from 4.952 R to 4.958 R—R being a revolution of the screw and equal to one mm. The distance δ between the surface of the liquid and that of the metal plates above was 4.09 cm. and the width of the slit between the plates was 10–11 mm. The distance between the crests A and B was twenty wave-lengths.

The experiments upon mercury were all made upon one fine, clean surface of pure distilled mercury, which retained its character well throughout the observations. It was protected from dust, etc., by the wooden frame R (Fig. 1), the condenser plates GG, and by paper, which was tacked about the frame and closed up the liquid almost completely. The distance δ between the surface and the plates

above was 3.41 cm., and the width of the slit 9 mm. The wave-length for the uncharged surface changed during the observations from 3.699 R to 3.679 R. The distance between the crests A and B was thirty wave-lengths.

The mean results of the observations, averaged according to agreement of potentials, are contained in Tables II. and III., and exhibited graphically in Figs. 6 and 7. The circles represent positive, and the crosses negative electrifications.

The effect on the wave-length is seen to be independent of the sign of the charge; and much greater in the case of water than in the case of mercury, as would be expected from the relative densities and tensions of the two. The motion of crest A is approximately twice that of B for the reason that its distance from the dipper was about twice as great—some 20 cms. Sometimes, with the water,

TABLE II.

Water. Positive Charges.						Water. Negative Charges.					
$10^{-3}\times$ Potential in volts.	Motion of crest A in revs. of screw.	No. of observations	$10^{-3}\times$ Potential in volts.	Motion of crest B in revs. of screw.	No. of observations	$10^{-3}\times$ Potential in volts.	Motion of crest A in revs. of screw.	No. of observations	$10^{-3}\times$ Potential in volts.	Motion of crest B in revs. of screw.	No. of observations
8.9	0.25	6	10.7	0.16	6	8.9	0.28	6	8.0	0.12	6
11.1	0.34	..	13.1	0.26	..	11.0	0.39	..	11.6	0.22	..
12.6	0.49	..	16.1	0.39	..	12.7	0.41	..	14.0	0.28	..
14.2	0.62	..	18.8	0.56	..	14.6	0.62	..	16.1	0.40	..
16.2	0.82	..	20.1	0.68	..	15.6	0.75	..	17.6	0.44	..
17.4	0.98	..	22.0	0.75	..	16.7	0.82	..	18.4	0.55	..
18.6	1.23	..	23.7	0.96	..	18.3	1.05	..	19.4	0.55	..
19.4	1.34	..	25.4	1.07	..	19.8	1.32	..	20.4	0.75	..
20.8	1.41	..	26.9	1.05	4	20.3	1.42	..	21.5	0.73	..
22.0	1.68	..				21.2	1.52	..	22.0	0.81	..
23.3	1.78	..				22.0	1.74	..	23.2	0.96	..
24.1	2.08	..				22.7	1.80	..	23.8	0.95	..
24.9	2.18	..				23.3	1.85	..	24.6	0.96	..
25.5	2.36	..				24.4	1.98	..	25.4	1.08	..
26.6	2.42	4				25.6	2.32	..	26.0	1.07	..
						26.0	2.10	..	26.6	1.12	..
						26.3	2.18	..	27.1	1.12	2
						26.8	2.30	..			
						27.1	2.55	4			

$\lambda = 4.96$ revolutions of screw.
Distance from liquid surface to metal plate above $= \delta =$ 4.09 cm.
B is the crest nearer the dipper; A, 20λ farther away.

TABLE III.

Mercury. Positive Charges.						Mercury. Negative Charges.					
$10^{-3}\times$ Potential in volts.	Motion of crest A in revs. of screw.	No. of observations	$10^{-3}\times$ Potential in volts.	Motion of crest B in revs. of screw.	No. of observations	$10^{-3}\times$ Potential in volts.	Motion of crest A in revs. of screw.	No. of observations	$10^{-3}\times$ Potential in volts.	Motion of crest B in revs. of screw.	No. of observations
11.1	0.10	4	11.6	0.06	4	10.7	0.09	4	12.5	0.06	4
15.2	0.17	..	16.6	0.11	..	13.3	0.11	..	16.7	0.08	..
16.5	0.15	..	18.5	0.14	..	14.7	0.16	..	21.0	0.14	..
19.0	0.29	..	20.9	0.16	..	18.1	0.24	..	22.8	0.15	..
20.2	0.24	..	23.0	0.19	..	18.8	0.22	..	23.5	0.16	..
20.8	0.30	..	24.8	0.20	..	19.7	0.24	..	24.1	0.19	..
22.0	0.38	..	26.0	0.21	..	20.3	0.27	..	24.8	0.14	..
23.6	0.41	..	27.1	0.26	..	21.0	0.31	..	25.9	0.18	..
24.9	0.45	..	27.7	0.28	..	21.8	0.32	..			
26.8	0.48	..	28.4	0.30	..	22.9	0.40	..			
27.4	0.54	..				23.2	0.39	..			
28.5	0.57	5				23.8	0.42	..			
						24.3	0.42	..			
						25.4	0.49	..			

$\lambda = 3.69$ revolutions of screw.
Distance from liquid surface to metal plate above $= \delta =$ 3.41 cm.
B is the crest nearer the dipper; A, 30 λ farther away.

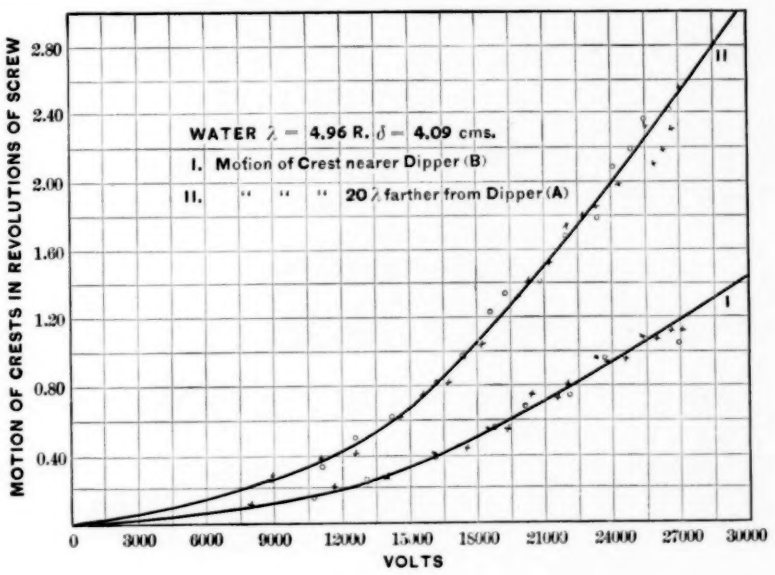


Fig. 6.

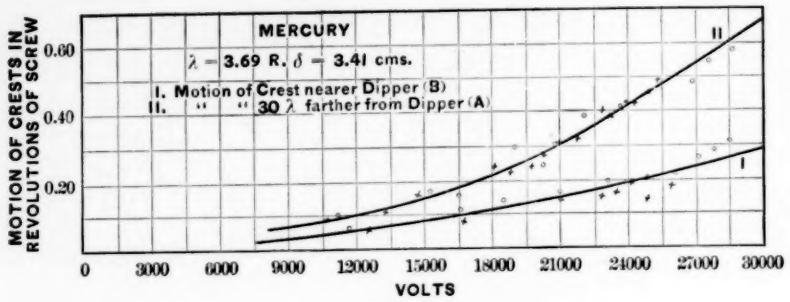


Fig 7.

the amplitude of the fork was increased while the motion of A was being measured, and then diminished again for B, in order to make the definition of both as good as possible. This was regularly done in the experiments on mercury, as the small effect on its wave-length made every help to accurate measurement indispensable.

From the curves in Figs. 6 and 7 and the values of λ (wave-length) already given were obtained the first five columns of Table IV., and the first three of Table V. σ (electric surface density) and σ^2 were computed from the formula for parallel plate condensers— $\sigma = \frac{V}{4\pi\delta}$, where δ if the distance between the plates and V is their potential difference in electrostatic units. T is the apparent surface tension (= true surface tension $- F(\sigma^2)$), given in percentage of its value for no electrification, and was obtained by the aid of Kelvin's formula. We have for an infinitesimal variation of λ, caused by a variation of the electrification

$$\frac{dT}{T} = 3\frac{d\lambda}{\lambda}\left\{\frac{1 - \frac{g}{3\pi\lambda^2 n^2}}{1 - \frac{g}{2\pi\lambda n^2}}\right\}.$$

The quantity in the parentheses is sensibly constant for such small variations of λ as occur in these experiments. In the case of mercury the maximum change in λ is so small—0.3 per cent.—that we may at once write, throughout the range of actual variation of λ,

$$T_0 - \Delta T = T_0\left(1 - 2.73\frac{\Delta\lambda}{\lambda}\right) = T.$$

In the case of water the diminution of λ reaches 1.35 per cent., and more accurate results are obtained by applying this formula in succession to each value of T. Thus

$$T_1 = T_0 - \Delta T_0 = T_0\left(1 - 2.92\,\frac{\Delta\lambda_1}{\lambda_0}\right),$$

$$T_2 = T_1\left(1 - 2.92\,\frac{\Delta\lambda_2 - \Delta\lambda_1}{\lambda_1}\right), \text{ etc.}$$

In this way the values of T, given in the final columns of the tables, were computed, with T_0 put equal to 100. In the curves of Figs. 8 and 9 the results contained in the tables are shown graphically. The apparent surface tension is given both as a function of the surface density of the electric charge and as a function of its square. The relation between the tension and the density of the charge is seen to be nearly parabolic, especially in the case of water.

Some rough measurements were also made upon a dense solution of sodium chloride and a similar change in its apparent surface tension observed.

A similar diminution of the apparent tension of water was observed with alternating charges. One terminal of the secondary of a large induction coil was connected to earth and the other to the liquid, and

TABLE IV.

Water.

Potential in volts.	$20 \times \Delta\lambda$.	$\Delta\lambda$.	$\Delta\lambda_{n+1} - \Delta\lambda_n$.	λ	σ	σ^2	T
0	0.000	0.0000	0.0000	4.96	0.00	0.00	100.00
9000	0.132	0.0066	0.0066	4.96	0.58	0.34	99.61
12000	0.240	0.0120	0.0054	4.95	0.77	0.59	99.29
15000	0.348	0.0174	0.0054	4.95	0.97	0.94	98.98
18000	0.554	0.0277	0.0103	4.94	0.17	1.37	98.37
21000	0.788	0.0394	0.0117	4.93	1.36	1.85	97.69
24000	1.046	0.0523	0.0129	4.92	1.56	2.43	96.94
27000	1.330	0.0665	0.0142	4.91	1.74	3.03	96.13

λ, $\Delta\lambda$, etc., in revolutions of screw.
Frequency of ripples = 64.4.

TABLE V.

Mercury.

Potential in volts.	$30 \times \Delta\lambda$.	$\Delta\lambda$	σ	σ^2	T
0	0.000	0.00000	0.00	0.00	100.00
9000	0.034	0.00113	0.70	0.49	99.92
12000	0.048	0.00158	0.93	0.86	99.88
15000	0.068	0.00227	1.17	1.37	99.83
18000	0.101	0.00337	1.39	1.93	99.75
21000	0.160	0.00533	1.63	2.66	99.61
24000	0.229	0.00763	1.87	3.50	99.44
27000	0.304	0.0101	2.09	4.37	99.25
30000	0.374	0.0125	2.33	5.43	99.08

$\lambda = 3.69$. λ and $\Delta\lambda$ in revolutions of screw.
requency of ripples = 64.4.

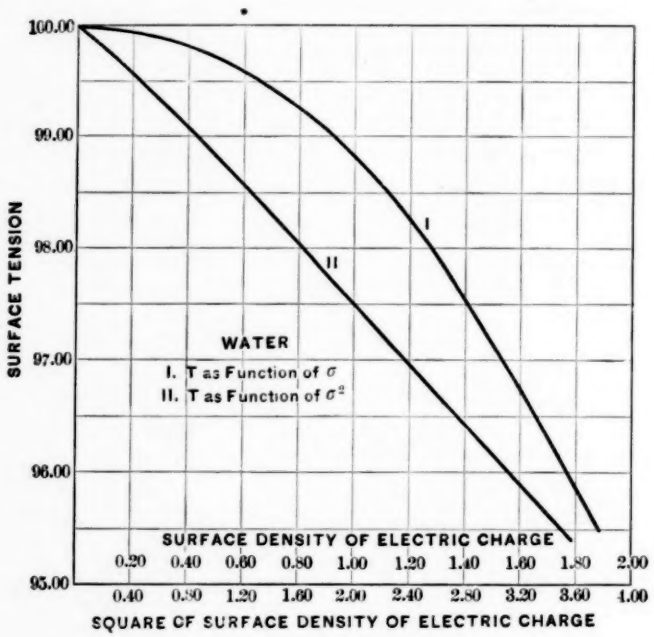


Fig. 8. The ordinate is the "apparent" surface tension.

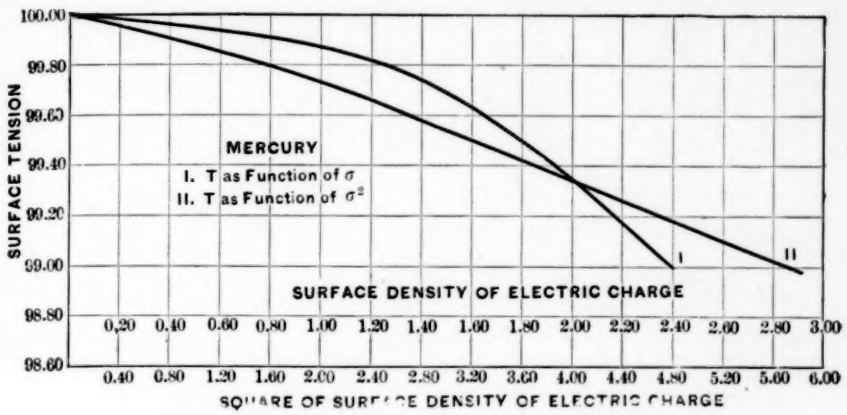


Fig. 9. The ordinate is the "apparent" surface tension.

the primary fed by an alternating current. In these circumstances the appearance in the telescope depends, as would be expected, on the phase relation between the charging of the surface and the intermittence of the light. This frequency of intermittence is twice the frequency of the forks, and the frequency of charging is twice the frequency of the alternator furnishing current, since positive and negative charges produce the same effect. The displacement of the crests varies from zero, when illumination takes place at the time of zero charge, to a maximum when illumination occurs at the time of maximum charge, either positive or negative. Of course, as the frequency of illumination is 128.8, only summational effects can be observed. With a frequency almost exactly double that of the intermittence of the light slow "beats" between the illumination and the charge were at one time observed, the bands moving to and fro in the field of the telescope. The rest of the time the crests appeared perfectly steady. With a frequency of charging of 110 the bands periodically underwent a rapid series of changes; breaking up into a number of similar bands moving in the field, then uniting in their original positions, only to break up again, the whole process repeating itself indefinitely. The explanation of these phenomena is obvious from what has been already said.

The results of this investigation, so far as they go, are in accord with the theory of Helmholtz given above. While the air above the

liquid surface is doubtless in a highly polarized state, such as obtains in Helmholtz's "Doppelschichten," all possibility of an electrolytic explanation of the effects observed seems excluded. The fact that the surface tension of the uncharged surface remains practically constant for hours, within which it has been repeatedly charged and discharged; the instantaneous nature of the effect, both in the case of steady potentials and in that of alternating potentials; and the identity of the results produced by positive and negative charges, all strongly negative any interpretation of the phenomena as other than electrostatic in origin. It is certain, however, that the diminution of wavelength observed on electrifying the surface is at least partially accounted for by the kind of electrostatic action whose effect is denoted by $F(\sigma^2)$ in the modified form of Kelvin's equation given above; and as this function has not yet been determined, it is possible that the whole effect observed is due to this cause, and no part of it to the forces tangential to the surface acting between the "elementary electric charges." If an effect of the latter kind exists, the results given above show that it must be much smaller than has hitherto been supposed; but, within experimental errors, the same for positive and negative charges. If this effect does not exist, the phenomena here described must be relegated to Group I. of the summary in the early part of this paper. It is remarkable in how many cases a diminution of surface tension would go to produce effects of the same kind as those actually produced by electrostatic and electrolytic causes.

To Professors Nichols and Merritt and to Mr. Homer J. Hotchkiss I am indebted for the effective interest which they have taken in this investigation throughout its progress, especially for their valuable suggestions.

THE PHYSICAL LABORATORY OF CORNELL UNIVERSITY
March, 1898.

ON THE FALL OF POTENTIAL AT THE SURFACE OF A METAL WHEN EXPOSED TO THE DISCHARGING ACTION OF THE X-RAYS.

By C. D. Child.

§1. If one causes an electric current to flow through the air from a point to a metallic screen placed opposite to it, it is found that the potential gradient determined by a Thomson water dropper increases noticeably in the neighborhood of the screen. This fact was brought to the attention of the present writer by Professor Warburg

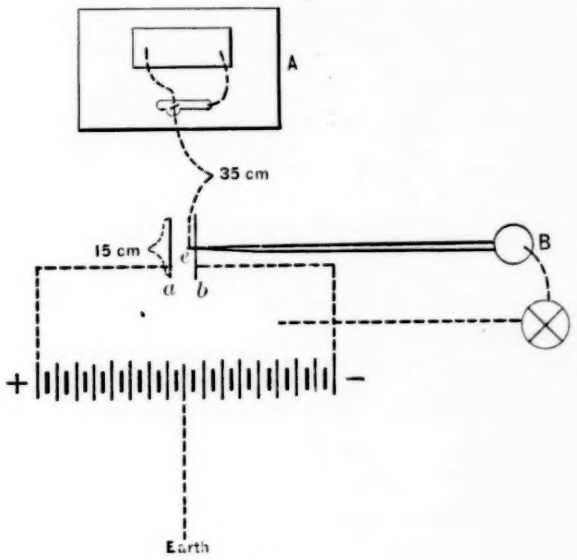


Fig. 1.

and it was suggested that an investigation be made in order to determine if a similar effect took place in the case of discharge produced by X-rays.

§2. The arrangement of the apparatus used is shown in Fig. 1. *A* is a lead box containing the induction coil and the X-ray tube. The rays pass through an opening 5 cm. in diameter. The distance from this opening to the tube is 5 cm. *a* and *b* are two charged plates, the difference of potential between them being kept

constant by means of a battery. The water flows through a tube ending in as fine a point as possible at *e*. The water in the beaker is connected to one pair of quadrants of a Thomson quadrant electrometer. The other two quadrants are connected to a point on the battery used having nearly the same potential as *e*. In this way a sensitive electrometer could be used without having too large deflections. The plates first used were aluminium 15 cm. in dia-

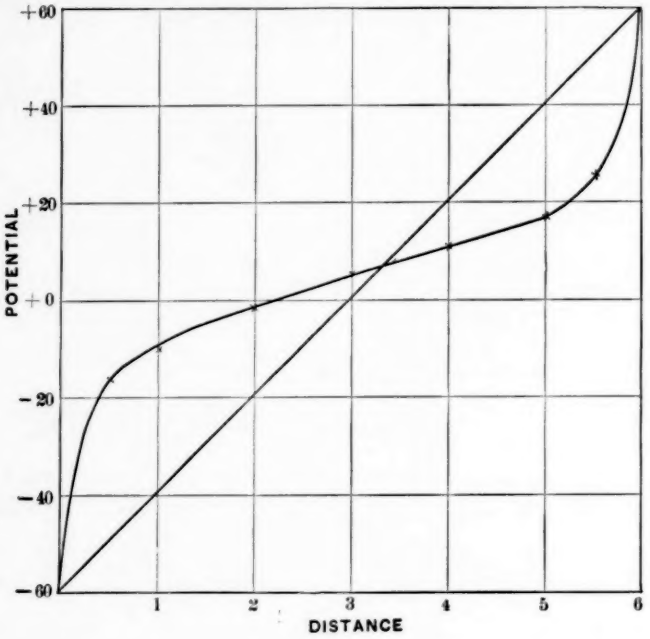


Fig. 2.

meter. The distance between them was 6 cm. The distance from the center of the plates to the tube was 35 cm. The difference of potential between the plates was 120 volts, the potential of *a* being +60 and that of *b* −60. All other bodies in the vicinity of the plates were kept at zero potential. It ought also to be stated that the potential between the plates was not always the same, even when all the conditions were kept as nearly constant as possible. This variation was due, no doubt, to the irregular action of the tube. However, the variation in several sets of observations was not more than two or three volts.

§3. The intensity of the rays was such that when the plates

were 2 cm. apart, the difference of potential between them 120 volts, and their average distance from the tube 25 cm., the current produced was 1.3×10^{-8} ampere. This was determined by measuring the rise in potential of the plate *b*, when it was in multiple with a capacity of 1 microfarad, the X-rays passing between the plates for 30 sec.

The fall in potential when no X-rays were passing was practically uniform, and it may be indicated by the straight line *cd* in Fig. 2. Table I. gives the potential in volts between the plates before and during the time the X-rays were passing between them, also the difference between these two sets of values.

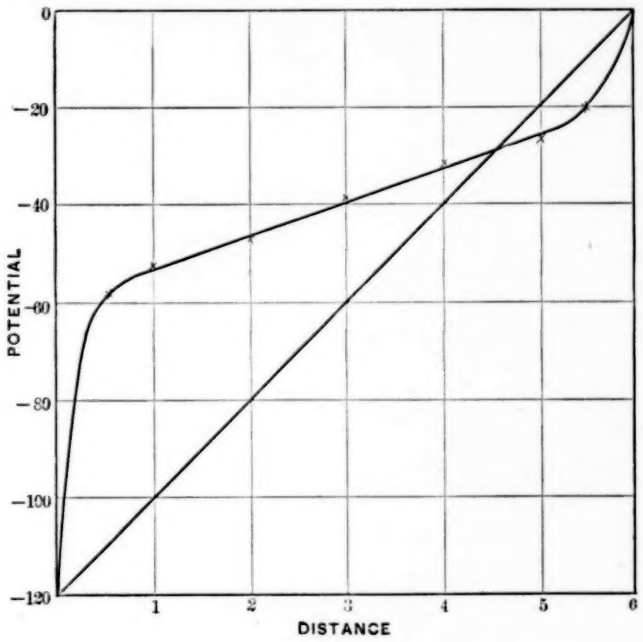


Fig. 3.

TABLE I.

Distance from Plate *b* to *c*.	Potential Without X-rays.	Potential with X-rays.	Difference in Potential.
.5 cm.	− 50	− 16.	34.
1 "	− 40	− 10.	30.
2 "	− 20	− 1.5	18.5
3 "	0	5.5	5.5
4 "	+ 20	11.	− 9.
5 "	+ 40	19.	− 21.
5.5 "	+ 50	25.5	− 24.5

Fig. 2 shows this relation graphically.

It will be seen that in general *the curve has the same form as that found when electricity passes through a rarefied gas.* The similarity is the greater inasmuch as the fall at the negative plate is greater than that at the positive.

§4. It was not possible to arrange the apparatus so that the potential of the surrounding bodies had no effect in the experiment. But this effect, though considerable, did not change the essential character of the results, as is shown by the following observations. These show the difference between the normal potential and that observed when X-rays are passing, first with a at zero potential and b at -120 volts and then with a at $+120$ volts and b at zero potential.

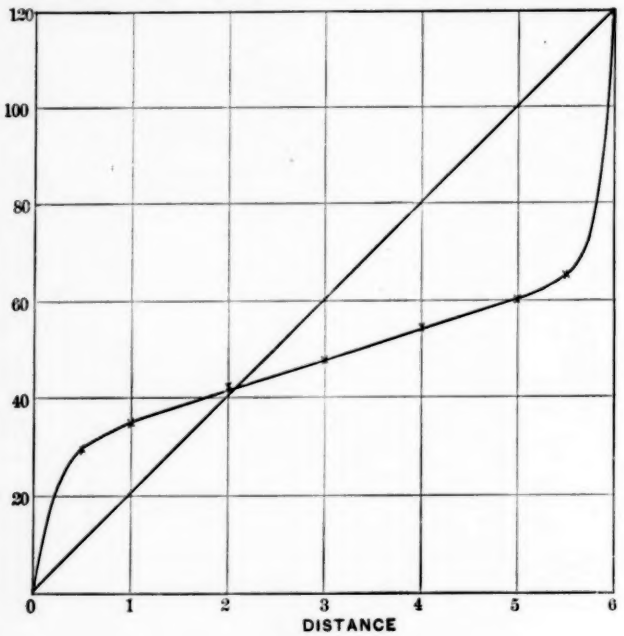


Fig. 4.

TABLE II.

Distance from Plate b to e.	Difference in Potential when a was zero.	Difference in Potential when b was zero.
.5 cm.	52	19.6
1 "	47.5	15.
2 "	32.7	2.
3 "	21.	−12.5
4 "	7.	−26.
5 "	− 7.	−40.
5.5 "	−10.	−45.

Figs. 3 and 4 show these relations graphically.

§ 5. The fall of potential at the plate was then studied under various conditions. The difference of potential between the plates was first varied. The distance *ab* was 6 cm. and *eb* 1 cm. The intensity of the rays was kept as nearly constant as possible. The difference between the normal potential at *e* and that observed when X-rays were passing is given in Table III. The plates were charged equally and oppositely, the surrounding bodies being at zero potential.

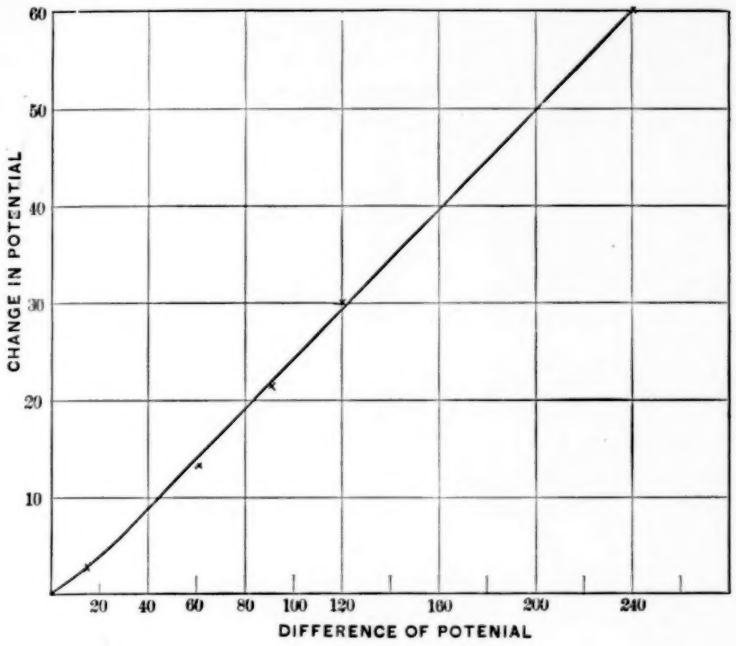


Fig. 5.

TABLE III.

Difference of Potential between *a* and *b*.	Change in Potential at *e*.
16 volts.	3.
30 "	6.25
60 "	13.3
90 "	21.6
120 "	30.
240 "	60.

Fig. 5 shows this relation graphically.

§ 6. In the next experiment the distance between *a* and *b* was

varied. The ratio $eb : ab$ was constant and equal to $\frac{1}{4}$, so that the potential at e when no X-rays were passing was always the same. The difference of potential between the plates was 120 volts. The intensity of the rays was kept as nearly constant as possible.

The results are given in Table IV. and are shown graphically in Fig. 6.

TABLE IV.

Distance *ab*.	Change in Potential at *e*.
2 cm.	4.7
4 "	10.8
6 "	16.1
8 "	18.7
12 "	21.6

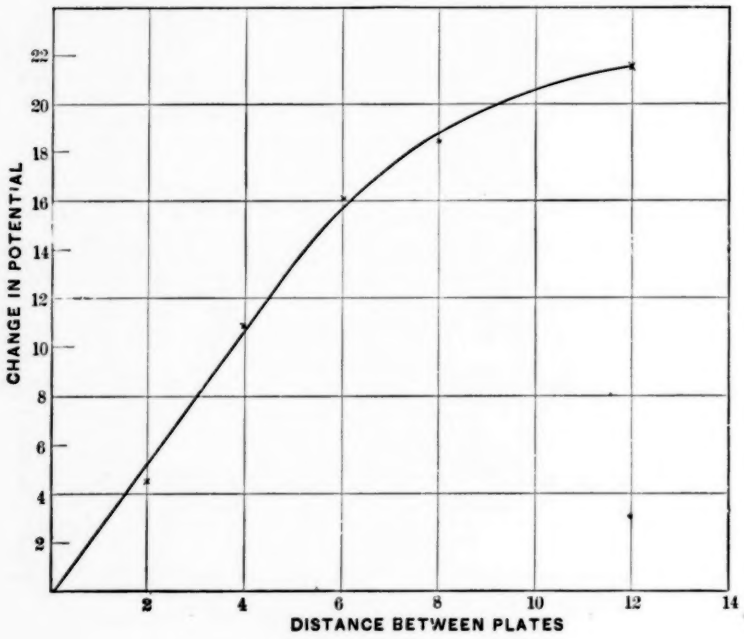


Fig. 6.

§ 7. Finally the difference between the plates and the difference of potential were kept constant, and the intensity of the rays was varied. The distance ab was 6 cm. and eb 1 cm. The difference of potential was 120 volts. The intensity of the rays was varied by placing pieces of tin foil between the tube and the plates. The in-

tensity was determined by measuring the amount of discharge that it would produce between two pieces of tin foil placed perpendicular to the rays, the distance between the pieces being 2 cm., and the difference of potential 120 volts.[1] The original intensity was taken as unity.

TABLE V.

Intensity of Rays.	Change in Potential at e.
1.	30.
.39	21.6
.1	11.6
.04	4.2

Fig. 7 shows this relation graphically.

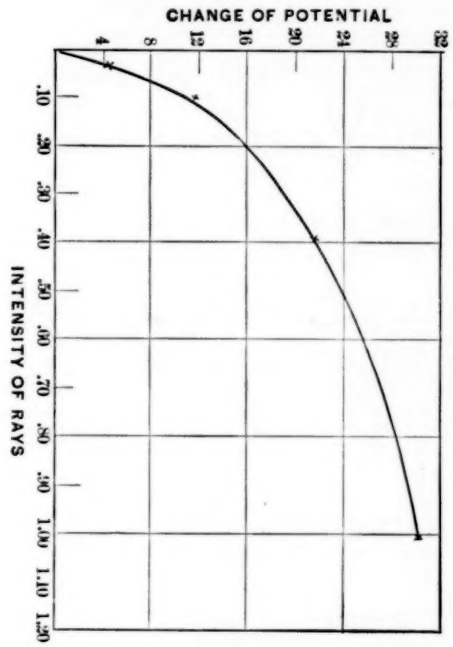


Fig. 7.

§8. When plates of other metals were used, the fall of potential

[1] There is, of course, the possibility that the rate of discharge is not proportional to the intensity of the X-rays, yet it is highly probable that it is. It has been shown by photographic measurements that the intensity of the rays varies practically inversely as the square of the distance from the tube. Thomson and McCelland have shown that the discharging power of the rays also varies inversely as the square of the distance, and this law was roughly tested and found to hold for the same pieces of tin foil that had been used to measure the intensity of the rays.

at the surface of the plates was found to be smaller, but when an opaque obstacle was so placed that the rays nearly filled the space between the plates without touching the plates themselves, no variation in the fall of potential with different metals could be detected. In other words, any variation in the fall of potential produced by varying the metal, depended on the fact that some of the rays struck the metal.

In order to study the effect of different metals to better advantage, the plates were, therefore, placed perpendicular to the rays. It was found, however, that the curve *cd* in Fig. 2 was entirely distorted, due to the fact that the conductivity of the gas near the tube was greater than that further from it. It was also impossible to use any metal on the side next the tube, other than aluminium or tin foil, because of the greater absorption by any others that were to be had.

First, however, an attempt was made to determine whether the fall of potential diminished or increased when the rays struck the plates normally instead of being parallel to them. The potential was found 1 cm. from the negative plate, first, when the negative plate was nearest the tube, and second, when it was farthest from it. By taking the average of these two measurements any effect due to the greater conductivity of the gas nearer the tube was largely eliminated. The change in potential 1 cm. from the negative plate was 14.1 volts, when the plate was nearest the tube, and 26.5 volts, when it was farthest from it. The average change in potential was 20.3 volts.

Of course, in both these cases there was a sheet of aluminium between the point where the potential was noted and the tube. When the plate was parallel to the rays and the rays passed through a piece of aluminium of the same thickness as the plate in the former case, the change in potential 1 cm. from the negative plate was 28.3 volts. The mean distance of the plates from the tube in both cases was 35 cm. It would thus seem that the fall of potential at the plates was diminished by allowing the X-rays to strike the plates normally. This fact was still more apparent when other metals were used.

In comparing the different metals the plate nearest the tube,

which we will call *a*, was always tin foil, and the plate *b*, the one farthest from the tube, was varied. All the plates were 15 c. in diameter. *c* is a point 1 cm. from *b*. The other conditions were the same as they were in the first experiment. The following results were obtained with the different metals.

TABLE VI.

Metal at *b*.	Change in Potential at *c* when *b* is Positive.	Negative.
Platinum	− 9	14
Lead	−11	17
Steel	−15	21
Zinc	−15	22
Copper	−15	22
Iron	−16	22
Tin foil	−16	23
German Silver	−15.5	23.5
Brass	−16	24.5
Aluminium	−17.5	26.5

The fall of potential depended to a very small extent upon the surface of metal, rust or dirt affecting the result slightly.

§ 9. We should expect the current between the plates to be least in the case of metals which show the greatest fall of potential at the surface. The rate of discharge with different metals has been investigated by Benoist and Hurmusescu,[1] but it seemed well to find the rate of discharge with the same metals that had been used in the previous experiments. The rate of discharge was determined by connecting an electrometer to the plate *b* for a short length of time, while a discharge was passing, and noting the deflection of the electrometer.

It has been already found that when the rays passed between the plates without touching them the fall of potential was independent of the kind of metal used. Similarly it was found that under the same conditions the rate of discharge between the plates was independent of the metal used.

But when the plates were perpendicular to the rays, the rate of discharge depended very largely upon the kind of metal. In the following experiments the distance between the plates was 1 cm., and the difference of potential was 120 volts. The plate *a* was

[1] C. R. 122, 779.

always of tin foil and changed positively. The average of a number of different observations is given.

TABLE VII.

Metal.	Deflection of Electrometer.
Platinum	16.6
Lead	15.8
Steel	13.9
Zinc	13.1
Copper	12.4
Tin foil	12.1
Iron	11.8
German Silver	11.8
Brass	11.6
Aluminium	9.8

It will be noted that, with the exception of tin foil, the order of the metals in this list is the same as that in Table VI., so that it is evident that in general *there is the largest rate of discharge from those metals which have the least fall of potential at the surface.*

The ratio between the current when the plate b was platinum and that when it was aluminium is seen in the above case to be 1.7. This was found to vary as the difference of potential was varied, as is shown in the following table. The distance between the plates and the intensity of therays were the same as before.

TABLE VIII.

Difference of Potential.	Deflection of Electrometer with b Aluminium.	Platinum.	Ratio of the Deflections.
8 volts.	5.3	6.7	1.26
18 "	7.5	13.3	1.77
30 "	8.4	15.	1.78
60 "	8.8	16.	1.82
120 "	10.	17.	1.70

It appears that it makes little difference what kind of metal is used when the difference of potential is small, but that the saturation current is reached sooner in the case of aluminium, so that when the potential is large, the rate of dischange depends largely on the kind of metal.

Next, the distance between the plates was varied, the plates being moved so that the average distance of the plates from the tube was constant. The difference of potential between the plates was 120 volts.

TABLE IX.

Distance Between the Plates.	Deflection of Electrometer with		Ratio of the Deflections.
	Aluminium.	Platinum.	
.5 cm.	8.5	15.2	1.79
1 "	9.3	16.6	1.79
2 "	10.2	18.	1.76
4 "	11.2	18.	1.61
8 "	10.5	15.4	1.46

It appears that the ratio between the rates of discharge in the two cases becomes slightly greater as the distance between the plates is decreased.

We thus see that, under certain circumstances, the rate of discharge produced by the X-rays depends largely upon the nature of the metal used, while, under other circumstances, the nature of the metal has little effect on the rate of discharge, and that when the rays do not touch the plate, it seems to have no effect. This fact, no doubt, accounts for the contradictory conclusions to which different observers have come regarding the variation of the rate of discharge with different metals.

§ 10. Benoist and Hurmusescu made the statement that the metals which absorb X-rays most, discharge electricity when under the action of X-rays most freely. It has not been possible to determine the absorbing powers of the metals used, nor have data been found for more than a few of the metals, but it is well known that in general the denser metals absorb X-rays most. It is, therefore, evident that in general the foregoing experiments substantiate the statement of Benoist and Hurmusescu, and they also show that there is the least fall of potential at the surface of the plates which absorb X-rays most. The fact that this fall of potential and the variation in the rate of discharge with different metals depends upon the striking of the metals by the rays, accord well with this view.

§ 11. These experiments show, therefore, first, *that there is a large fall of potential at the surface of the metal when discharge is caused by X-rays;* second, *that this fall of potential is diminished by allowing the X-rays to strike the plate*, and third, *that it is diminished to the greatest extent in the case of metals which absorb X-rays most.*

It is of interest to compare these results with those found by Elster and Geitel[1] in the case of discharge of electrified bodies by

[1] Wied. Ann., 61, 445.

ultra-violet light. It was apparent from their experiments that the absorption by the metal of the light plays an important part in the discharge of the electricity, while the experiments here described show that the absorption by the metal of the X-rays likewise plays an important part.

It is evident that the preceding experiments throw doubt upon any determination of the velocity of the ions which is based upon the assumption that the fall of potential between the plates is uniform. It is possible to make a determination of this velocity by an investigation of the length of time that discharge continues after the X-rays cease, and, in fact, the velocity thus determined was found to be much greater than that found by Rutherford.[1] However, it has not yet been possible for the writer to attain the desired accuracy in this determination, and it, therefore, seemed better to postpone the publication of any results until the work could be done more accurately.

In conclusion, I wish to express my thanks to Professor Warburg and also to Dr. Kaufmann for their kind aid and assistance.

PHYSICAL INSTITUTE OF BERLIN UNIVERSITY,
March, 1898.

[1] Phil. Mag., Oct., '97.

AN EXPERIMENTAL DETERMINATION OF THE PERIOD OF ELECTRICAL OSCILLATIONS.[1]

By Arthur Gordon Webster.

[The following research was carried out in August, 1893, and the manuscript has been out of my possession until within a few weeks. In the meantime extended articles on the same subject have been published by Tellqvist[1] and Seiler.[2] As however the methods and instruments here employed are quite different from theirs, and as the oscillations studied by me were of much shorter period, and free from the influence of the dielectric in mica condensers, and furthermore as the method also lends itself to a determination of the important quantity "**v**," in a way not depending on determinations of the Ohm, I feel warranted in publishing it unchanged.]

The question whose solution is here attempted concerns the experimental verification of the formula giving the period of the oscillations of an alternating current arising when a charged condenser is discharged through a circuit containing self-induction. Any such investigation will necessarily take as a point of departure the classical researches of von Helmholtz[3] on Electrical Oscillations, published over twenty years ago, and continued in an exhaustive manner by Schiller.[4] It consequently remains only to measure absolutely the quantities involved, according to the system of electrical units with which we are now familiar.

Such is the scope of the present investigation. The method, though similar to those of von Helmholtz and Schiller, makes use of different instruments designed and constructed for the purpose, and the experimental arrangement is quite different. In the experi-

[1] This investigation was awarded the Elihu Thompson Prize. See Science, Vol. 1, p. 190 and p. 294; also Nature, Vol. 51, p. 324. (Ed.)

[2] Tellqvist, Wied. Ann. 60, p. 248, 1897. Ueber die Oscillatorische Entladung von Condensatoren.

[3] Seiler, Wied. Ann. 61, p. 30, 1897. Ueber Oscillationen bei der Ladung von Condensatoren und ihre Anwendung zur Bestimmung des Selbstpotentials beliebiger Leitersysteme.

[4] Helmholtz, Ueber Electrische Oscillationen, Ges. Werke, I., p. 531.

[5] Schiller, Pogg. Ann. 152, p. 535, 1874.

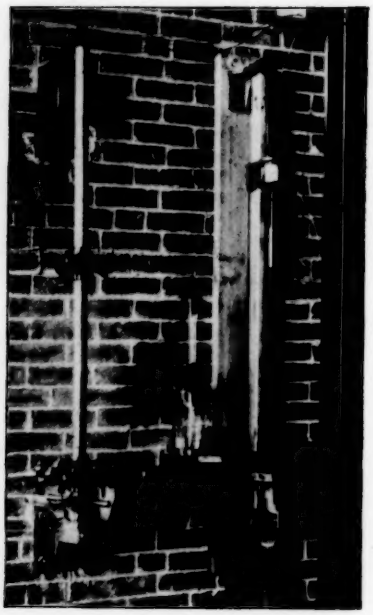
ments of Schiller a primary current flowing through an induction coil was broken by a falling pendulum, and the potential-difference at the ends of the secondary coil connected with a condenser were measured at subsequent instants by means of a quadrant electrometer. I have also measured the potentials by means of an electrometer, but have avoided the use of an induction coil, in order to simplify the conditions. It is known that the layers of a coil of wire in which a current flows act as a condenser, of which it is impossible to calculate the capacity, and in the case of a long induction coil of fine wire this capacity may be considerable in amount. This accordingly introduces an element of doubt, which it is desirable to eliminate. Now any coil of wire having perceptible self-induction will in some degree possess this undesirable capacity, but it may be made small by properly choosing the coil.

Coefficients of self-induction, as well as capacities, are ordinarily measured in terms of resistances and times, so that Messrs. Ayrton and Perry were led to propose the name secohm for what is now generally known as the henry. In order to avoid this dependence upon the value of the ohm, I wound a coil, the coefficient of self-induction of which could be calculated directly from its geometrical measurements. For the condenser, for a similar reason, and in order to be free from effects due to electric absorption, dielectric hysteresis, and conductivity, I made use of an air-condenser whose capacity is known in electrostatic measure from its geometrical measurements. In this manner, besides verifying the formula, I obtain a method of determining "**v**," the ratio betwen the two electric units.

The Instruments.

The pendulum interrupter[1] of von Helmholtz is well adapted to its purpose, but I determined to attempt to improve it in several respects. In the first place, the falling body, the pendulum, exposes a large and irregular surface to the air, so that the air resistance must be considerable. Secondly, the time-value of the readings of the instrument is invariable, and the absolute value is difficult to determine. The determination of Schiller by Pouillet's galvano-

[1] Vid. Schiller, loc. cit.



metric method is inconvenient and doubtful. I accordingly determined to replace the pendulum by a body falling freely, of such a shape that the air resistance would be negligible, or at least constant, so that the time-values could be directly determined from the height of fall, which might be varied. The instrument (Fig. 1), which will be referred to as *the interrupter*, consists of a solid bar B (firmly secured to the wall by two heavy brackets, in which it could be placed vertically by screws), carrying three carriages, two of which slide vertically upon the bar, while the third is screwed to it near its lower end. The upper carriage, A, carries an electromagnet, M, which supports the projectile, P. The proper design of the latter to insure an accurately straight fall was the most difficult point of the whole research, and it was many months before the interrupter was deemed sufficiently perfect for its work. The lower end of the core of the electro-magnet was of the shape shown in Fig. 1*a*, the conical cavity in the end having a more acute angle than the cone upon the end of the projectile, in order that the projectile upon breaking the current might fall directly without sliding down sidewise upon the cone, and thus being diverted from its proper course. The arrangement first tried of a sharp end to the core and a hollow end for the projectile never succeeded in giving a straight fall.

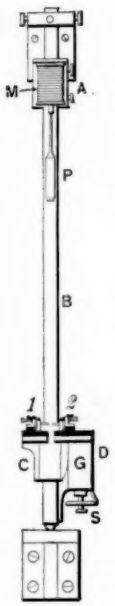


Fig. 1a.

Fig. 1b.

The projectile (shown half size in Fig. 1 b[1]) was accurately turned of steel and its lower portion was hollow and filled with mercury. In its end was inserted a cylindrical piece of agate. The projectile weighed in the neighborhood of five hundred grams. At the lower part of its course the projectile struck off the two contact-levers 1 and 2, the former being held by the

[1] Since replaced by a longer projectile, shown in the photograph.

carriage *C*, secured in place by three set-screws, the latter being movable on a slide *G* by means of a micrometer screw *S*, upon the fixed carriage *D*. The contact levers, Fig. 1*c*, were made of hardened steel, as light as was consistent with rigidity, and pivoted upon axes turning upon pointedscrews *a*, finished by grinding. Contact was made with platinumpointed screws *c*, clamped with nuts, carried upon standards standing upon a separate piece of ebonite, *e*. In order that the levers might be held against the contacts with the smallest possible, invari able force; instead of springs small permanent magnets, *m*, were used, cut in the form of screws, so that their distances from the levers might be adjusted to give just the proper pressure. The whole contact apparatus stood upon a base of ebonite, *e'*. The projectile was caught, after passing through the apparatus, in a wooden cylinder lined with velvet and chamois skin. The contact-levers after being struck off, were caught in spring clips *S*, to prevent them from rebounding and to connect them to earth. The results at tained by the performance of this apparatus may be judged by the fact that on several days the position of the zero, determined by the method described later, did not vary by an amount corresponding to an interval of one five-millionth of a second.

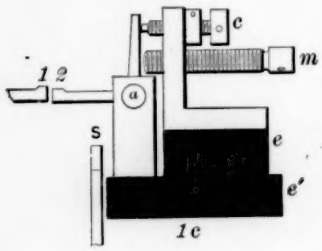


Fig. 1c.

The condenser was a plate-condenser, exactly similar to that used and described by Himstedt[1] in his determination of "**v**," and consisted of two steel plates half a meter in diameter and seventeen millimeters thick, constructed for me by Messrs. Breithaupt und Sohn in Cassel, Germany. They were ground plane, nickel-plated and polished on one side, and separated by sets of three small cylinders cut from plates of plane and parallel glass. These cylinders before use were cleaned in alcohol and distilled water in which they were left for some time, and then carefully dried with filter-paper, as described by Himstedt.[2]

[1] Himstedt, Eine Bestimmung der Grösse "v," Wied. Am. 29, p. 560.

[2] The condenser was supported upon ebonite cones resting upon three glass legs, about 60 cm. from the floor, and over two meters from any object.

The capacity was calculated by the formula given by Kirchhoff,[1] and amply confirmed experimentally by Himstedt, by comparison with a guard-ring condenser.

$$K = \frac{r^2}{4d} + \frac{r}{4\pi}\left[1 + \log_e \frac{16\pi r(b+d)}{d^2} + \frac{b}{d}\log_e \frac{b+d}{b}\right]$$

where r is the radius and b the thickness of the plates, d their distance apart.

The electrometer was a modified Mascart-Kelvin quadrant electrometer, reconstructed in the workshop of the university. The suspension was a long quartz fiber of such a diameter as to give a period of twelve seconds for a half swing. The whole instrument was carried in a thin glass jar partly filled with sulphuric acid and serving as Leyden jar. From the acid the needle was charged by means of a platinum wire just dipping into the acid, and furnishing damping enough to bring the needle to rest in four or five minutes, the nearest interval, accordingly, at which observations could succeed each other. For a reason given later, it was essential that the equality of the coefficients of induction between the needle and quadrants should be capable of accurate adjustment, and hence beside the coarse adjustment possible by sliding one quadrant in and out by hand, the opposite quadrant was given a slow motion by means of a micrometer screw, as in the Kelvin electrometer. The cylinder carrying the torsion head was also revolved by a micrometer screw, in order to bring the needle to precisely the right angular position. The torsion head, and hence the needle hanging from it, was adjustable in all three coördinates, and a last adjustment was to be found in the levelling screws supporting the instrument.

The instrument was observed by a telescope and scale at a distance of three and a-half meters, and the sensibility was such that with the needle charged by fifty cells of water-battery, the deflection given by seven Leclanché cells was 105 mm.

The self-induction was furnished by a coil of copper wire insulated with a double winding of silk, wound in a groove two centimeters wide in a circular frame of wood, constructed of a great many

[1] Kirchhoff, Monatsber. and Berl. Acad., 1877, p. 144.

pieces of wood laid up and glued together, and afterwards carefully turned. The frame was made several years ago and well seasoned.

The resistances were taken from a box by Elliott Brothers, and were found by calibration to be correct to one part in five thousand. The battery employed consisted of forty-five small storage cells, insulated with paraffin, fractions of whose electromotive force were taken off at pleasure.

MEASUREMENT OF THE GEOMETRICAL QUANTITIES.

All length measurements are referred to a meter brass bar, constructed by the Société Genèvoise, which was compared with the national prototype at the Office of Weights and Measures at Washington. The diameter of the condenser plates was measured by direct comparison with the above meter on a comparator, two diameters of each plate being measured. Thus was found for one plate,

$$D = 49.94888 \text{ cm.}$$

and for the other, $D = 49.97193$ "

giving as a mean, for the radius,

$$r = 24.9802 \text{ cm.}$$

For the thickness was found, as a mean of four measurements of each plate,

$$b = 1.6855 \text{ cm.}$$

Three sets of glass cylinders were used, giving three different capacities. Their heights were measured by the screw of a spherometer constructed by the Société Genèvoise, and compared in the part used with one centimeter of the above-mentioned meter. The method of measuring the cylinders and of calibrating the screw may be of interest, and was as follows: The instrument in which the screw was placed was an application of Michelson's refractometer, and is shown in Fig. 2. Two slides, *A* and *B*, moved accurately between gibbs upon an iron base. The slide *A* was moved by the spherometer screw *S*, whose point pressed against a flat projec-

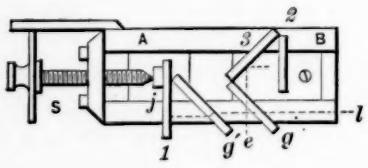


Fig. 2.

tion, j, rising from the slide. The other slide was generally clamped by a tightening screw. The two slides carried the two plates g and g', and the three mirrors 1, 2 and 3. A monochromatic or white light was placed at l, and the eye at e. When the adjustments were made with monochromatic light, fringes were seen in g, and as the screw S was turned one fringe passed a marked point for every half wave-length advanced by the screw. When the fringes were brought to that point where they were most distinct (maximum visibility) white light was substituted for monochromatic, and on turning the screw slowly, the few colored fringes, which alone exist in white light, were soon found. The central white fringe makes an admirable zero position, and on taking a reading was always brought to a fixed mark on the mirror. To measure a cylinder the fringes were first brought to the marked position, and a reading taken on the screw head, which read to thousandths of a millimeter. The cylinder to be measured was then inserted between the point of the screw and the projecting piece j, and the screw turned back until the central fringe again fell on the mark. A second reading was then taken, and the difference of the two gave the thickness of the cylinder. Each cylinder was measured four times, being turned end for end each time, and the zero point was found before and after each set.

The results for all three different thicknesses are given.

$d_1 = 6_1$ mm. +

	a	b	c
Readings	225.0 μ	219.0	225.0
	223.5	218.0	223.0
	223.0	221.0	223.0
	222.0	219.0	223.5
Means	223.4	219.2	223.5

Mean of three cylinders 222.0

Zero reading 80.0

$d_1 = 6.1420$ mm.

$d_2 = 3.8 +$

	a	b	c
Readings	016.0 μ	013.0	011.0
	016.0	012.0	011.0
	016.0	012.0	011.0
	016.0	012.0	011.0
Means	016.0	012.25	011.0

Mean of three cylinders 0133
Zero reading 1590

$d_2 = 3.8543$ mm.

$d_3 = 1.9 +$

	a	b	c
Reading	108.0	109.0	115.0
	108.0	109.0	114.5
	108.0	110.0	114.0
	108.0	111.0	114.0
Means	108.0	109.75	114.37

Mean of three cylinders 110.71
Zero reading 161.50

$d_3 = 1.9492$ mm.

The screw was calibrated by measuring the thickness of one of the shortest cylinders in six successive places on one centimeter of the screw, the zero being shifted by moving the slide B the required distance each time, and reclamping it. The error of the screw was found too small to take into account. Finally, the calibrated part of the screw was compared with a centimeter of the meter bar.

To the capacities as calculated from these data were added small corrections for the dielectric constant of the glass cylinders, 5.2 mm. in diameter, for which purpose the dielectric constant of the glass was assumed to be 5. The values of the capacities obtained were :

$$K_1 = 275.56 \text{ cm.}$$
$$K_2 = 428.145$$
$$K_3 = 826.50$$

The coefficient of self-induction of the coil was calculated from the formula given by Lord Rayleigh.[1]

$$L = 4\pi a n^2 \left[\log_e \frac{8a}{r} + \tfrac{1}{12} - \tfrac{4}{3} \left(\vartheta - \frac{\pi}{4} \right) \cot 2\vartheta - \tfrac{1}{3} \pi \operatorname{cosec} 2\vartheta \right.$$

[1] Rayleigh, On the Absolute Value of the *B. A.* Unit, Phil. Trans., 1882, Pt. II.

$$- \tfrac{1}{6} \cot^2 \vartheta \log_e \cos \vartheta - \tfrac{1}{6} \tan^2 \vartheta \log_e \sin \vartheta \Big]$$

where n is the number of turns, a is the mean radius of the coil, r the diagonal of the rectangular cross-section, and ϑ the angle made by r with the plane of the coil. A calculation by the formula (58) p. 170, Mascart and Joubert, *Traité d'Electricité et Magnétisme*, confirmed this result.

The diameter of the coil was measured by the cathetometer and the circumference of the groove before and after winding was measured with a steel tape. The mean radius was thus found to be $a = 22.878$ cm.

The width of the channel being 2 cm., and the depth of the winding 1.766 cm.,

$$r = 2.668 \text{ and } \vartheta = 48^\circ\ 32'\ 44''$$

There were 572 turns, so that the final result is

$$L = 3.1811 \times 10^8 \text{ cm.}$$

The resistance of the coil was found to be 55.95 ohms.

The time value of one division of the head of the micrometer screw of the interrupter was calculated from the height of fall of the projectile and the value of the acceleration of gravity. It was intended to make a chronographic determination of this element, but the time set for the completion of the investigation did not permit of this. The accuracy of this means of determination is, however, sufficient for the present purpose.

The acceleration of gravity was calculated from the formula

$$g = 980.6056 - 2.5028 \cos 2\lambda - .000003\ h$$

with $\lambda = 42^\circ\ 17'\ 20''$, $h = 16000$ cm. giving

$$g = 980.321 \text{ cm./sec.}^2$$

The height of fall was found by means of a cathetometer by the Société Genèvoise secured to the wall close by the interrupter. The micrometer of the upper telescope was set upon the sharp edge of the piece of agate in the end of the projectile when hanging from the electro-magnet, the micrometer of the lower telescope being at the same time set upon the upper edge of the lever of fixed height, which was struck by the projectile, the distance was then

measured upon a meter bar attached to the brackets supporting the interrupter. This distance was found to be invariable during the course of the experiments and was

$$h = 73.405 \text{ cm.}$$

The time of traversing the vertical distance Δh is $\Delta t = \frac{\Delta h}{\sqrt{2gh}}$ if we may neglect Δh in comparison with $4h$, but as this was not the case in all the experiments, the term involving the second power of Δh in the development of $t = \sqrt{\frac{2h}{g}}$ was made use of, giving

$$\Delta t = \frac{\Delta h}{\sqrt{2gh}}\left(1 - \frac{\Delta h}{4h}\right).$$

The factor of correction thus obtained was $\left[1 - \frac{d}{1321200}\right]$ where d is the reading of the drum a of the micrometer. The micrometer screw, by the well-known firm of Brown and Sharpe, was compared with the standard centimeter and found to be correct with a pitch of one-half millimeter, and the drum was divided into 225 parts. Accordingly τ, the value of one division, is

$$\tau = .00000058577 \text{ sec.},$$

or 1707.1 divisions equals one one-thousandth of a second.

EXPERIMENTAL ARRANGEMENTS.

Several different arrangements of the electrical connections were successively tried, and they will be described in turn, although they were eventually abandoned for the last one devised. In all the figures (Fig. 3) B denotes the battery, R the resistance of the coil of self-induction L, K the condenser, K_0 the electrometer, 1 and 2 respectively the upper and lower contact levers, and E the earth connection, which was very thoroughly made by soldering in several places to a gas pipe.

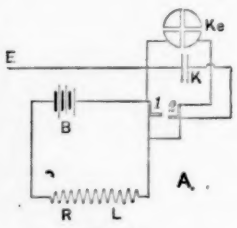


Fig. 3 *A*.

In the first and simplest arrangement A, one pair of quadrants of the electrometer is connected permanently to the earth. As long

as the contact 1 is closed, the condenser and electrometer, which is connected in parallel with it, are short-circuited, both plates being at the potential of the earth. When 1 is broken, the battery begins to charge the condenser through the resistance R and self-induction L, and the potential is represented at any subsequent time t by

$$\mathbf{v} = \mathbf{v}_0 \left(1 - e^{-\frac{Rt}{2L}} \cos \sqrt{\frac{1}{KL} - \frac{R^2}{4L^2}}\, t\right)$$

At any time t the contact 2 is broken, the electrometer is insulated, and the needle swings out under the influence of the charge that it had at potential $\mathbf{v}$ at the time t. The time t is vanishingly small in comparison with the time of swing of the electrometer needle, which being, therefore, at rest during the whole time t has a constant capacity K_0, which is to be added to that of the condenser K. After the electrometer is insulated and begins to swing, to be sure its capacity changes, but its charge remains constant, and experiment shows that the deflections are proportional to the potentials of the quadrant at the instant when contact 2 was broken. The method of verifying this statement by a calibration will be described hereafter.

The objections to this method are: first, that it is not a zero method, and although the curve of potential (a) might be carefully observed, the determination of the time of oscillation, which is the quantity required, would be difficult. Furthermore, in order to observe any considerable number of oscillations, the damping must be small, and the resistance R must be so small that the battery would be likely to polarize. Lastly, for the same reason, the resistance of the contact 1, which cannot be assumed to be absolutely zero, would have a ratio to R that could not be neglected, and hence the electrometer would not stand exactly at zero before the break.

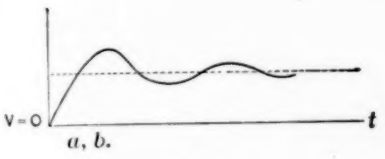


Fig. 4.[1]

It now occurred to me that since the deflections of the electrom-

[1] The letters a, b, c, d, in Figs. 4 and 5, refer to the arrangements A, B, C, D.

eter are proportional to the differences of potentials of the two pairs of quadrants, a zero method might be obtained by suddenly connecting the other pair of quadrants, which in A is permanently connected to earth, to a source of constant potential, and making this equal to the potential $\mathbf{v}$ expected at the end of the time t. To this end it was first necessary to ascertain whether the needle would really stand at rest when the quadrants were both electrified to the same potential other than zero. In order to attain this result it was necessary to adjust the positions of the quadrants and needle very carefully.

Let us consider the conditions of equilibrium of a slightly unsymmetrical electrometer. If the potentials of the two quadrants and the needle be $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ respectively, the charges e_1, e_2, e_3, we have

$$\begin{aligned} e_1 &= q_{11}\mathbf{v}_1 + q_{12}\mathbf{v}_2 + q_{13}\mathbf{v}_3 \\ e_2 &= q_{12}\mathbf{v}_1 + q_{22}\mathbf{v}_2 + q_{23}\mathbf{v}_3 \\ e_3 &= q_{13}\mathbf{v}_1 + q_{23}\mathbf{v}_2 + q_{33}\mathbf{v}_3 \end{aligned}$$

where under the ordinary conditions of use, namely, small deflections ϑ,

$$\begin{aligned} q_{11} &= c_1 + a_1\vartheta & q_{12} &= c_4 \\ q_{22} &= c_2 - a_2\vartheta & q_{13} &= c_5 - a_1\vartheta \\ q_{33} &= c_3 + (a_1 - a_2)\vartheta & q_{23} &= c_6 + a_2\vartheta \end{aligned}$$

The energy of the system is

$$\begin{aligned} W &= \tfrac{1}{2}\Sigma e\mathbf{v} = \tfrac{1}{2}q_{11}\mathbf{v}_1^2 + \tfrac{1}{2}q_{22}\mathbf{v}_2^2 + \tfrac{1}{2}q_{33}\mathbf{v}_3^2 \\ &\quad + q_{12}\mathbf{v}_1\mathbf{v}_2 + q_{23}\mathbf{v}_2\mathbf{v}_3 + q_{13}\mathbf{v}_3\mathbf{v}_1 \\ &= \tfrac{1}{2}\mathbf{v}_1^2[c_1 + a_1\vartheta] + \tfrac{1}{2}\mathbf{v}_2^2[c_2 - a_2\vartheta] + \tfrac{1}{2}\mathbf{v}_3^2[c_3 + (a_1 - a_2)\vartheta] \\ &\quad + c_4\mathbf{v}_1\mathbf{v}_2 + \mathbf{v}_2\mathbf{v}_3[c_6 + a_2\vartheta] + \mathbf{v}_1\mathbf{v}_3[c_5 - a_1\vartheta]. \end{aligned}$$

The couple tending to turn the needle is,

$$\theta = \frac{\partial W}{\partial \vartheta} = \tfrac{1}{2}a_1\mathbf{v}_1^2 - \tfrac{1}{2}a_2\mathbf{v}_2^2 + \tfrac{1}{2}\mathbf{v}_3^2(a_1 - a_2) + \mathbf{v}_3[a_2\mathbf{v}_2 - a_1\mathbf{v}_1],$$

and if $\mathbf{v}_1$ and $\mathbf{v}_2$ are made equal, $\mathbf{v}$,

$$\theta = \tfrac{1}{2}\mathbf{v}^2[a_1 - a_2] + \mathbf{v}_3\mathbf{v}[a_2 - a_1] + \tfrac{1}{2}\mathbf{v}_3^2(a_1 - a_2)$$

Thus we see that unless $\mathbf{v} = \mathbf{v}_3$ the needle will not remain at rest unless $a_1 = a_2$. In order to make the adjustment, we may make $\mathbf{v} = 0$, and take readings with $\mathbf{v}_3$ first zero and then with a large

value. In order to thoroughly test the above general formula two commutators were arranged, so that a given potential $\mathbf{v}$ might be applied either positively or negatively to either quadrant, while the other was connected to earth. Four different deflections were thus obtained whose differences gave the relative values of a_1 and a_2. The quadrants were then moved until these values were equal, which was finally given a last test by charging them to the same high potential, and then reversing its sign. The difference of the last two readings is equal to $2(a_1 - a_2)\mathbf{v}\mathbf{v}_3$ and could be made imperceptible when $\mathbf{v}$ was ninety volts, while $\mathbf{v}_3$ was the potential due to one hundred water elements.

This point having been established, the arrangement shown in *B* was tried. In order to change the potential of the second quadrants immediately from 0 to $\mathbf{v}$ they were connected to the lever of a relay *r*, whose back contact was connected to earth while the front contact was connected to a variable point *P* between the resistances R_1 and R_2 whose potential was $\mathbf{v}_0 \frac{R_1}{R_0 + R_2}$ where $\mathbf{v}_0$ is the electromotive force of the battery. The circuit of the relay was closed by the lever 1 being struck into the clip *c*, and so little time was lost that in a test where the quadrants 1 were connected directly to 1′ instead of to 2′ and charged instantaneously through a small resistance without self-induction to the full value $\mathbf{v}_0$, which was made the potential of *P* also, the deflection on dropping the projectile was not over one scale-division, whereas the potential applied to one quadrant alone would have driven the mirror off the scale.

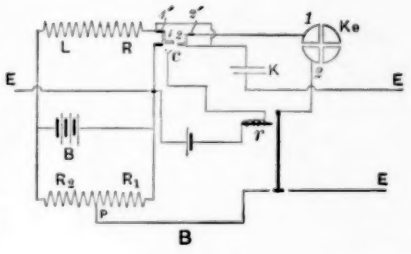


Fig. 3 *B*.

This arrangement promised to work well, and was a zero method, but one circumstance induced me to modify it; namely, after the contact 2 is broken, the lever 2 remains at the potential $\mathbf{v}_0$ and acts inductively upon the support 2′, thus affecting the charge of the quadrant. Hence it was desirable to put the lever 2 to earth imme-

diately after breaking. To this end the arrangement was modified as shown in *C*. The quadrants and condenser were both charged at the beginning instead of at the end, and while quadrant 1 and the condenser discharged through R, the potential of quadrants 2 was shifting from $\mathbf{v}_0$ to zero. It must be observed that this shifting did not take place until after the contact 2 was broken, so that the capacity of the electrometer K_0 was not disturbed. The clip c' put the lever 2 to earth immediately after breaking, so that there was now no disturbing action upon the electrometer. By this arrangement was for the first time obtained a state of things where a zero potential might be observed, which is certainly to be desired on the score of simplicity, while at the same time the course of the phenomenon corresponds exactly to that proposed in stating the question, viz.: the *dis*charge of a condenser by an oscillating current. This arrangement suggested the arrangement that was finally adopted, which enabled me to dispense with the complication of the relay. In the final form, *D*, the two quadrants and condenser are originally at the same high potential, which is however only the fraction $\frac{R_1}{R_1 + R_2}$ of the whole potential V_0. By the introduction of a resistance of one thousand ohms at R_2 the influence of the resistance of the contact 1 is eliminated. On breaking the contact 1 the quadrants 2 discharge instantaneously through the resistance R_1 (its time-constant of discharge KR_1 being only one twenty-five-millionth of a second) while the condenser and quadrant 1 discharge through the resistance $R + R_1$. By turning the micrometer screw the points of zero potential are easily obtained,

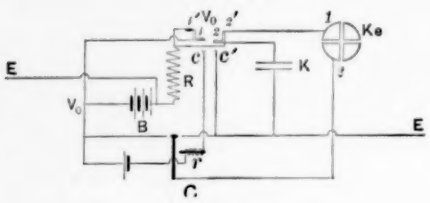


Fig. 3 *C*.

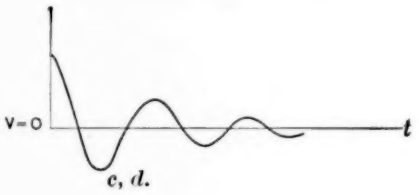


Fig. 5.

one-fifth of a division being sufficient to cause a change of sign of the deflection.

The arrangement shown in *E* was that used in finding the zero point, namely that at which both contacts were broken together. The condenser was removed, so that the time-constant was negligibly small, and the screw moved until the deflection on dropping just ceased.

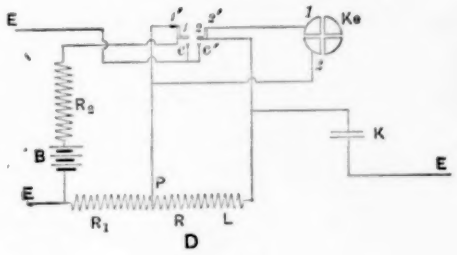


Fig. 3 *D*.

The method was so sensitive that the zero could be determined to one-tenth of a drum-division, a change of one division being sufficient to change the deflection from zero to off the scale.

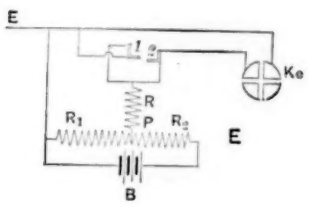


Fig. 3 *E*.

The same arrangement was used in making the calibration above spoken of, the contacts being separated so far that there was no doubt that the fuli potential $\mathbf{v}_0$ was attained. The potential used was then varied by moving the point *P*, and the deflections were found, as stated, proportional to the potentials. This calibration was, however, no longer needed in the method adopted.

The Experiments.

Before beginning the final experiments the electrometer was taken down and refilled with fresh, pure sulphuric acid, which had been boiled with ammonium sulphate. The electrometer was set up on a shelf between the cathetometer and the interrupter, and the short wire connected with the screw 2′ was shielded by a tin tube connected to earth (not shown in the photograph). The adjustment of the quadrants was next made, for which a potential $\mathbf{v}$ of eighty or ninety volts was employed. The leakage of both sets of quadrants was then determined, and found to be too small to influence the results. The zero point of the interrupter was determined before and

after each set of oscillation experiments. The times of oscillation were determined for the electrometer and coil alone, and for the three different condensers used. In observing, a zero point not too near the origin was first observed, and then the one next it, and it

$a_1 + nT_1 = l_1$, to determine T_1 and a_1.

n	l_1'	l_1 Obs.	l_1 Calc.	Obs—Calc.	
10	761.5	761.00	761.508	— .508	Giving $T_1 = 67.756$ with a mean error of $\mu_1 = \pm .0473$.
11	829.5	828.92	829.071	— .151	
21	1509.5	1507.60	1506.634	+ .966	
31	2187.5	2183.70	2184.198	— .498	
		$a_2 + nT_2 = l_2$.			
2	235.2	235.14	233.595	+ 1.545	$T_2 = 77.695$ mean error $\mu_2 = .0587$
3	311.2	311.10	311.290	— .190	
12	1009.5	1008.66	1010.550	— 1.890	
22	1789.8	1787.20	1787.505	— .305	
32	2570.5	2565.30	2564.460	+ .840	
		$a_3 + nT_3 = l_3$.			
4	532.1	531.40	532.516	— 1.116	$T_3 = 100.478$ mean error $\mu_3 = .192$
5	637.2	636.85	633.090	+ 3.760	
15	1642.5	1640.30	1637.874	+ 2.426	
26	2745.0	2739.10	2743.131	— 4.031	
		$a_0 + nT_0 = l_0$.			
1	104.0	103.98	102.428	+ 1.552	$T_0 = 42.2006$ mean error $\mu_0 = .243$
2	145.5	145.47	144.629	+ 0.841	
8	394.0	393.85	397.833	— 3.983	
18	822.0	821.43	819.839	+ 1.591	

was in each case noted whether the potential was rising or falling, so that the number of the oscillation might be determined without doubt. Zero points ten, twenty or thirty half-oscillations farther on were then observed, and the results all combined by the method of least squares. The results follow. In the first column is given n, the number of the zero-point, counting from the first which occurs as zero. In the second column is the observed reading l' in drum divisions, from an arbitrary zero. In the third column are the

readings l reduced by the factor given above. In the fourth column are the readings calculated from the value derived from the reduction by least squares, and in the next column are the residual errors.

The differential equation giving the current or potential in terms of the time is

$$L\frac{d^2x}{dt^2}+R\frac{dx}{dt}+\frac{\nu^2}{K}x=0$$

where K is measured electrostatically, giving as a solution,

$$x=Ae^{-\frac{R}{2L}}\cos\left(\sqrt{\frac{\nu^2}{KL}-\frac{R^2}{4L^2}}\cdot t-\varepsilon\right)$$

and for the time of a half period

$$T=\frac{\pi}{\sqrt{\frac{\nu^2}{KL}-\frac{R^2}{4L^2}}}$$

In the present investigation, the resistance and capacity are so small that the second term of the denominator is quite negligible in comparison with the first, and we may use the simpler expression

$$T=\frac{\pi}{\nu}\sqrt{KL}$$

Putting[1] $a=\frac{\nu^2T^2}{\pi^2L}$ we have four equations to determine a and K_0.

$$\tau T_n=\frac{\pi}{\nu}\sqrt{(K_n+K_0)L} \text{ or } \frac{\nu^2\tau^2}{\pi^2L}T_n^2-K_0=K_n$$

As there is no reason for assuming that there was any difference in the accuracy of the determination of the three different capacities, but as the times of oscillation were determined with different degrees of precision, weights were assigned to the four equations in accordance with the mean errors given above, and values of K_0 and a determined.

$a\,(67.756)^2-K_0=275.56$
$a\,(77.695)^2-K_0=428.145$ giving $K_0=182.02$
$a\,(100.478)^2-K_0=826.50$ $\log a=9.0002922-10$
$a\,(42.2006)^2-K_0=0.$

From a, ν was calculated as

[1] If T is measured in drum-division, and τ is the time value of one division.

$$\frac{\pi}{\tau}\sqrt{aL} = \nu = 3{,}0259 \cdot 10^{10} \text{ cm./sec.}$$

The fact that the value of "v" obtained should be in error by a large fraction of one per cent. is not surprising, as the investigation was not undertaken with a view to the determination of this quantity, but only of verifying the formula for the period of the oscillation, as proposed by the committee. This verification I hope to have satisfactorily given. The discrepancy in the value obtained is probably due to inaccuracy in the measurement of the quantities involved in the coefficient of self-induction of the coil, which was not of the best possible form for accurate measurement. In order to determine "v" accurately by this method, it would only be necessary to use several different coils of larger cross-section, and in addition to make a chronographic determination of the time-constant of the interrupter. Apparatus for this purpose has been designed and constructed, and, although the time at my disposal did not permit of further experiments in the time allowed, this will probably be carried out in a future investigation.

Clark University, April, 1898.

NOTE.

William A. Rogers. Professor William A. Rogers, who died March 1, 1898, at Waterville, Maine, was born at Waterford, Conn., on the 13th of November, 1832. His early schooling was completed at Alfred Academy in western New York. He entered Brown University, and was graduated with the class of 1857, after which he returned to Alfred as a teacher in the academy which had prepared him for college. In 1859 he became Professor of Mathematics and Astronomy in Alfred University, with which institution he was connected until 1870. In 1866 the title of his chair was changed to that of Professor of Industrial Mechanics. In 1870 he severed his connection with that institution and was appointed assistant in the Harvard College Observatory. In 1875 he was promoted to an assistant professorship at Harvard, which position he held until 1886, when he was called to the chair of physics and astronomy in Colby University. This chair he occupied up to the time of his death. Had he lived one month longer Professor Rogers would have become the director of the new Babcock School of Physics at Alfred University, a school the establishment of which was largely due to his own efforts and for the equipment of which he himself presented apparatus valued at ten thousand dollars.

Professor Rogers' career as a teacher at Alfred suffered a variety of interruptions. One year was spent, in further preparation for his work, at the Harvard Observatory. More than a year was given to the direct service of his country, to which end, in 1864, he enlisted in the United States Navy where he remained until the close of the Civil War. In 1865–66 he built and equipped for the college an astronomical observatory After receiving the appointment to the George B. Rogers Professorship of Industrial Mechanics, he spent several months in special preparation for his duties in the Sheffield Scientific School at New Haven.

The ten years spent at Alfred University were years of activity and of preparation rather than of scientific productiveness. It was not until 1869, just before his resignation from the faculty of that institution, that he published his first paper. It may be said indeed that Professor Rogers' fruitfulness as a scientific man began with his appointment to the Astronomical Observatory at Cambridge. From 1873 until the day of his death scarcely a year passed which did not see some new contribution from his pen to our knowledge in the domains of astronomy or of physics. His earlier work was devoted chiefly to the former science and his services in the laborious zone work, in which the members of the Harvard College Observatory participated, may perhaps be considered the most important task of his life.

About 1880 he became interested in the problem of the accurate comparison of the yard and meter, and for nearly twenty years he devoted his time chiefly to improvements in the methods of measuring standards of length. Incidental to this problem was the study of the microscope, the dividing engine, the comparator and the interferometer, and to the mechanics of these instruments he made many contributions.

In 1873 Professor Rogers became a member of the American Academy of Arts and Sciences and in 1881 an honorary fellow of the Royal Microscopical Society of London. In 1895 he was elected to the National Academy of Sciences. He joined the American Association for the Advancement of Science in 1866 and for more than thirty years he was a constant attendant at the meeting of this Association and an active participant in its work. His was the unique distinction of serving as chairman of a section at three different meetings, namely as Vice-President of Section A in 1882 and again in 1883, and as Vice-President of Section B in 1894. He was President of the American Microscopical Society in 1887.

Professor Rogers' scientific activity remained unabated to the very end of his life. His untiring devotion to science and the indefatigable patience with which he pursued the most laborious investigations were proverbial. He had an especial fondness for work the success of which depended upon those qualities, and the unflagging enthusiasm with which he carried it on was a source of wonder and admiration to all his contemporaries.

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"On the Periodic Errors of the Right Ascension observed between 1858 and 1871." Read April 14, 1874. *Proc. Am. Acad. of Arts and Sciences*, IX., 127.

"New Elements of Brunhild (123), with Ephemeris for the Opposition, 1875." *Astronomische Nachrichten*, LXXXV., 241. Kiel, 1875.

"Death of Professor Joseph Winlock, Director of Harvard College Observatory." Id., LXXXVI., 113. Kiel, 1875.

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2. "Studies in Metrology." Id., Vol. 18, p. 287, 112 pp.

3. "On the Reduction of the Different Star Catalogues to a Common System." 15 p. *Proceedings of the Am. Acad. of Arts and Sciences*, 1882–3.

"A Study of the Problem of fine Rulings with Respect to the Limits of Naked-eye Visibility and Microscopic Resolution *Am. Monthly Microscopical Journal*, September, 1882.

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"On the Reduction of Different Star Catalogues to a Common System." 4 p. *Sidereal Messenger*, 1883. p. 204.

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"The Microscope in the Workshop." 5 p. *Proceedings of the Am. Microscopical Society.*

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